

Problem Set #3

due Monday, February 16, 2004

- doCarmo, section 1.5, # 2,9,14
- Let $\alpha(s)$ be a regular curve, parameterized by arclength, such that $\kappa(s) \neq 0$ and $\tau(s) \neq 0$ for all s .

(a) Prove that if α lies on the sphere of radius r , centered at p , then

$$\frac{\tau}{\kappa} = \left(\frac{\kappa'}{\tau\kappa^2} \right)'$$

(b) Prove that the center of the sphere, p , satisfies

$$p = \alpha(s) + \frac{1}{\kappa(s)}N(s) + \frac{\kappa'(s)}{\tau(s)\kappa^2(s)}B(s)$$

for all s .

(c) Prove the converse of part (a).

- Find a minimal set of first-order, linear differential equations which are equivalent to the Frenet–Serret equations for a curve in \mathbb{R}^3 . (Hint: You will need at least three equations.)