MAT 142
Quiz #6

March 10, 2005

Intructions: You have 15 minuntes for this quiz. You may not use any books, notes, or calculators. Justify every statement that you make. If you wish to use a theorem proved in class, state the theorem precisely. Good luck.

Given a function, $f(x)$, and a real number, a, we define a sequence, a_k, by

$$a_0 = a \quad a_{n+1} = f(a_n)$$

so that

$$a_0 = a \quad a_1 = f(a) \quad a_2 = f(f(a)) \quad a_3 = f(f(f(a)))$$

and so on. Prove that if the sequence converges and $f(x)$ is continuous, then the limit, L, satisfies the equation $f(L) = L$.

Hint: Compute $\lim_{n\to\infty} a_{n+1}$ in two different ways.

(A sequence defined in this manner is called a *dynamical system*. Professor Scott Sutherland is an expert on dynamical systems and will be giving an introductory talk to the Math Club this evening at 7:00 pm.)