MAT 141
Problem Set #4

due in recitation on September 30 or October 1, 2004

1. Apostol, section I 4.4, # 1 a–c, 3.
2. Apostol, section I 4.7, # 1, 8, 12
3. Let $F(n)$ denote the n–th Fibonacci number; that is, $F(1) = F(2) = 1$ and $F(n) = F(n-1) + F(n-2)$ for $n \geq 3$. Let $\phi = \frac{1+\sqrt{5}}{2}$ be the golden mean.
   (a) Show that $\phi^2 = 1 + \phi$ and that $(1-\phi)^2 = 2 - \phi$.
   (b) Prove (by strong induction) that $F(n) = \frac{\phi^n - (1-\phi)^n}{\sqrt{5}}$
4. Evaluate each of the following sums
   (a) $\sum_{i=1}^{n} (i+3)^2$
   (b) $\sum_{i=4}^{n} (i^2 - 9)$
   (c) $\sum_{i=1}^{n} i^4 - (i-2)^4$