MAT 141
Problem Set #12

due in recitation on November 23 or 24, 2004

1. Prove that if $f(x)$ is bounded on $[a, b]$ and discontinuous at only finitely many points of $[a, b]$, then $f(x)$ is integrable on $[a, b]$.
2. Assume that $f(x)$ is continuous on $[a, b]$. You are asked to defend the statement "$\lim_{x \to p} f(x) = f(p)$". Given a challenge, ϵ, describe a strategy for finding your response, δ, *that does not depend on* p. (Hint: You will need to use the small span theorem.)
3. Consider the function

$$g(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$$

Prove that $g(x)$ is differentiable at $x = 0$. Compute $g'(0)$.