MAT 614 FALL 2002: TOPICS IN ALGEBRAIC GEOMETRY

THE TOPOLOGY OF ALGEBRAIC VARIETIES AND MAPS

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OFFICE HOURS: Mo, 10-11 am, Math Tower P-143, Fr, 10-11 am, Math Tower P-143, Fr 2-3 Math Tower 3-115 (for this last hour if I am a bit late, please wait, I'm coming back from class).

CONTENTS: The main goal is to overview some of the topological and geometric methods needed to study algebraic varieties and maps. We will prove a version of the so-called Decomposition Theorem. This is the most general result known about the homology of maps between algebraic or Kähler varieties. It ties the topology of a map to a fundamental invariant of the target space: intesection cohomology.
Intersection cohomology is Goresky-MacPherson's generalization of singular cohomology to singular spaces. Its basic property is that it satisfies Poincaré Duality. Cohomology does not.

We will cover various aspects of the following topics. Hodge Theory and the topology of algebraic varieties: classical and "mixed." The language and properties of derived categories. Poincaré-Verdier Duality. Intersection cohomology. Stratified spaces and maps (after Mather-Thom-Whitney). The Decomposition Theorem. Relations with L^2-cohomology and algebraic cycles.

There will be strong emphasis on concrete examples from topology and complex geometry to see how the methods work in concrete situations.

HOMEWORK AND EXAMS: There will be no homeworks and no exams. However, there will be plenty of suggested exercises.

TENTATIVE SYLLABUS Likely, some items will be omitted or just briefly mentioned. References will be given. Many of the items below will only be surveyed. Key examples will be discussed. Meaningful exercises will be assigned. Complete proofs will be given only for selected items and for the theory of semismall maps. These are maps that appear in a variety of contexts in algebraic geometry, representation theory and mathematical physics. They are a special yet interesting case of maps. Much of the general machinery that will be covered in the course is put to use to understand these maps to which a great deal of classical geometry can be generalized. Students are going to be asked to lecture on selected items. I will assist them in preparing the lecture.

- Hodge Theory for complex manifolds, Poincare' Duality, Kodaira-Serre Duality.
- Kaehler manifolds, Kaehler identities, the Hodge decomposition, the Lefschetz decomposition, the Hard Lefschetz Theorem, the Hodge-Riemann Bilinear Relations, Hodge structures, mixed Hodge structures, polarizations.
- Sheaves, complexes, injective resolutions, cohomology, higher direct images, derived functors.
- Derived categories, the six operations, Poincare'-Verdier duality.
- Stratification theory of spaces and maps, the local structure of analytic spaces, the "local structure" of complex analytic maps, constructible sheaves, constructibility of higher direct images, the cohomological dimension of affine/Stein spaces.
- Line bundles/divisors/hypersurfaces, Bertini's Theorem, first Chern class, the Weak Lefschetz Theorem, the Kodaira Vanishing Theorem, pencils, monodromy, local systems.
- Intersection cohomology, the Decomposition Theorem of Beilinson-Bernstein-Deligne-Gabber.
- Borel-Moore homology, refined cup-products, the attaching triangle and interpretation.
- Semismall maps, the Lefschetz-Hodge-Kodaira package and the decomposition theorem.

REFERENCES. I have no particular textbook in mind. Some of the material to be presented is new and simplifies previous treatments and results in the subject. You can look at the following three papers to get a feeling for: the subject and its history [1]; the state of the art in 1982 [2]; what kind of tools will be developed in part of the course [3].

[1] S. L. Kleiman, "The development of intersection homology theory," in *A century of mathematics in America, Part II*, 543–585, Amer. Math. Soc., Providence, RI, 1989.

[2] R. MacPherson, “Global questions in the topology of singular spaces,” in *Proceedings of the International Congress of Mathematicians, Vol. 1, 2 (Warsaw, 1983)*, 213–235, PWN, Warsaw, 1984.
[3] M.A. de Cataldo, L. Migliorini, “The Hard Lefschetz Theorem and the topology of semismall maps,” available at http://www.math.sunysb.edu/~mde/publications.html

SPECIAL NEEDS. If you have a physical, psychiatric, medical, or learning disability that may affect your ability to carry out the assigned course work, please contact the office of Disabled Student Services (DSS), Humanities Building, room 133, telephone 632-6748/TDD. DSS will review your concerns and determine, with you, what accomodations are necessary and appropriate. All information and documentation of disability is confidential.