MAT 514 – MIDTERM

The maximum score is $200 = 100 + 100$.

NAME :

This test consists of two parts.

Part I consists of 20 questions. They are for the most part either true-false or yes-no questions. You should circle the right answer. The answer should be complemented by *very short* explanations. Each answer will generate up to 5 points for a total maximum of 100 points.

Part II consists of 5 problems. Problem C) will generate up to 40 points. The remaining ones up to 15 points. The total maximum is 100 points.

You must briefly justify all the answers.
You must use the space assigned.
Do not attach other sheets.
Write so that I can read: few short sentences should do.
You can quote a theorem from the the textbook if you want: in that case give the theorem number.
I suggest that you first write the answers on another sheet, correct them and *then* write the final answer. The final anwer must be in **pen**.
You can use any book you want.
You should not consult anybody.

PART I : 20 questions ; 100 points maximum

1) Let $f(x)$ be the function defined as follows:

$f(x) = \frac{1}{x^2}$, if $0 < x \leq 10$
and
$f(0) = 1$.

Is $f(x)$ bounded on the interval $[0, 10]$?

YES NO

2) Every bounded function is Riemann integrable.

TRUE FALSE

3) If f is a bounded function, then $|f|$ is a bounded function.

TRUE FALSE

4) Let f be a bounded function on $[a, b]$. Assume that $|f|$ is Riemann integrable on $[a, b]$. Then f is necessarily Riemann integrable on $[a.b]$.

TRUE FALSE

5) If f_1 and f_2 are bounded functions on $[a, b]$ and $f_1 + f_2$ is Riemann integrable on $[a, b]$, then f_1 is Riemann integrable over $[a, b]$.

TRUE FALSE

6) Is $f(x) = x^2$ Riemann integrable on $[-100, \pi]$?

YES NO

7) If f is not continuous on $[a,b]$, then f is necessarily not Riemann integrable on $[a,b]$.

TRUE FALSE

8) Let $f(x)$ be a Riemann integrable function on $[a,b]$ and c be a constant. Is $c\,f(x)$ Riemann integrable on $[a,b]$?

YES NO

9) Is

$$\int_0^3 (x^2+1)\sqrt{4+sin^2x}\,dx \geq 6\ ?$$

YES NO

10) If f is monotonic on $[a, b]$, then f is necessarily continuous on $[a, b]$.

TRUE FALSE

11) If f is monotonic on $[a, b]$, then f is necessarily Riemann integrable on $[a, b]$.

TRUE FALSE

12) Let f be a Riemann integrable function on $[0, 1]$. Then there is necessarily a point $c \in [0, 1]$ such that $f(c) = \int_0^1 f(t)\, dt$.

TRUE FALSE

13) Let f be a Riemann integrable function on $[a, b]$. Define a new function on $[a, b]$ by setting $F(x) = \int_a^x f(t)\, dt$. Is $F(x)$ Riemann integrable on $[a, b]$?

YES NO

14) Let f be a bounded function on $[a, b]$. Assume that you have a partition P of $[a, b]$ such that $U(P, f) = L(P, f)$. Is f Riemann integrable?

YES NO

15) If $P = \{x_0, \ldots, x_n\}$ is a partition of $[a, b]$, then we necessarily have $\Delta x_i = (b-a)/n$.

TRUE FALSE

16) Let f be a bounded function on $[a,b]$.
Assume that $\overline{\int} f\,dx = 3$ and $\underline{\int} f\,dx = 2$.
Then f is necessarily integrable and $2 \le \int_a^b f\,dx \le 3$.

TRUE FALSE

17) Write down one refinement of the partition

$$P = \{2, 2.1, 2.2, 2.3, 2.4, 2.5\}$$

of the interval $[2, 2.5]$.

18) If f and g are Riemann integrable on the interval $[a,b]$, then it is necessarily true that $\int_a^b f(x)g(x)\,dx = (\int_a^b f(x)\,dx)(\int_a^b g(x)\,dx)$.

TRUE FALSE

19) Let f be a Riemann integrable function on $[a, b]$. Let G be a differentiable function on $[a, b]$ such that $G' = f$. Then $G = \int_a^x f(t)\,dt$.

TRUE FALSE

20) Let f be a Riemann integrable function on $[a, b]$. Is the function $F(x) = \int_a^x f(t)\,dt$ continuous?

YES NO

PART II : 5 problems ; 100 point maximum

A) Prove that the following function is Riemann integrable on the inteval $[5, 6]$:

$f(x) = x, \qquad$ if $5 \leq x < 5.5$ and if $5.5 < x \leq 6$
and
$f(5.5) = -11.$

Note that you are not being asked to compute the integral.

B) Calculate the following integral

$$\int_{\pi/2}^{\pi} \sin x \cos x \, dx$$

C) Let $f(x)$ be a continuous function on $[a, b]$. Assume that $f(x) \geq 0$ for every $x \in [a, b]$. Assume that $\int_a^b f(x)\, dx = 0$.

Prove that $f(x) = 0$ for every $x \in [a, b]$.

D) Calculate

$$\int_{\sqrt[3]{2\pi}}^{\sqrt[3]{(5/2)\pi}} t^2 \cos t^3 \, dt$$

E) Calculate the derivative of the following function

$$T(x) = \int_{x}^{x+x^2} t \, dt$$