

**MAT 319 SPRING 2008
MIDTERM II**

!!! WRITE YOUR NAME, STUDENT ID BELOW !!!

NAME :

ID :

THERE ARE SIX (6) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK!!!

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS NO PHONES

ON YOUR DESK: ONLY test, pen, pencil eraser.

1		40pts
2		40pts
3		40pts
4		40pts
5		50pts
6		40pts
Total		250pts

!!! WRITE YOUR NAME, STUDENT ID AND LECTURE N. BELOW !!!

NAME :

ID :

LECTURE N.

1. State and prove the Cauchy Convergence Criterion.

2. In each case provide an example or state that it is not possible to do so and explain why.

2a Give one example of a sequence that is monotone, bounded and properly divergent.

2b Give one example of a sequence that is positive, bounded with two subsequences converging to 0 and 1, respectively.

2c Give one example of a sequence that is bounded and admits a divergent subsequence.

2d Give one example of a sequence that is unbounded and it admits a bounded subsequence.

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3. Give the definition of a Cauchy sequence and use the definition to show that

$$(\pi^n)$$

is not a Cauchy sequence.

Definition:

Proof that (π^n) is not a Cauchy sequence:

4. Complete the following sentences by inserting the words *convergent* or *divergent* (do not explain why):

4a. $\sum 1/\sqrt{n}$ is

4b. $\sum 1/n^2$ is

4c. $\sum (-1)^n (1/\sqrt{n})$ is

4d. $\sum (-1)^{n+1} (1/n^p)$ with $p > 1$ is

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5. Compute the two limits:

(5a)

$$\lim_{n \rightarrow \infty} \frac{(2 + \frac{1}{n})^n}{2^{n+2}}$$

(You may use without proof that the Euler number $e = \lim(1 + 1/n)^n$.)

(5b)

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{x - x^2}$$

(You may use without proof that, if $\lim f = c \geq 0$, then $\lim \sqrt{f} = \sqrt{\lim f}$.)

6. Show, using (ϵ, δ) , that for every $c \in \mathbb{R}$

$$\lim_{x \rightarrow c} x^3 = c^3.$$

