1.1

1. (a) $(3,1,2)$ parallel to $(6,4,2)$

iff $\exists t \in \mathbb{R}$ s.t. $t(3,1,2) = (6,4,2)$

Then $3t = 6$ implies $t = 2$, and $t = 4$.

This is impossible

2. Denote by v_A the vector starting from the oringin to the point A.

Then the equation of the line through two points A and B will be:

$$x = x(t) = v_A + t(v_B - v_A)$$

(a) $v_A = (3,-2,4)$, $v_B = (-5,7,1)$

So, $x = (3,-2,4) + t\big((-5,7,1) - (3,-2,4)\big) = (3-8t, -2+9t, 4-3t)$.

3. With the same notations above, the equation of the ~~line~~ plane through three points A, B, C will be:

$$x = x(t,s) = v_A + s(v_B - v_A) + t(v_C - v_A).$$

(a) $v_A = (2,-5,-1)$, $v_B = (0,4,6)$, $v_C = (-3,7,1)$

So, $x = (2,-5,1) + s(-2,9,7) + t(-5,12,2)$.

~~4. Let $0 = (a_1, a_2)$ and $x = (x_1, x_2)$~~ in $\mathbb{R}^2$, The axiom

$x + 0 = x$ implies that $\begin{cases} x_1 + a_1 = x_1 \\ x_2 + a_2 = x_2 \end{cases}$ for $\forall x_1, x_2 \in \mathbb{R}$

~~So $a_1 = a_2 = 0$~~

4. $\exists$ vector 0 s.t. $\forall x$, $x + 0 = x$. Write $0 = (a_1, a_2)$, $a_1, a_2 \in \mathbb{R}$

We must have $\begin{matrix} x_1 + a_1 = x_1 \\ x_2 + a_2 = x_2 \end{matrix}$ $\forall x_1, x_2 \in \mathbb{R}$.

By the cancelation property in $\mathbb{R}$: $a_1 = a_2 = 0$.

6. The line segment from (a,b) to (c,d) is parameterized by

$$x(t) = (a,b) + t\big((c,d) - (a,b)\big) \quad , \; t \in [0,1]$$

The midpoint should correspond $t = \frac{1}{2}$, and that is

$$x\left(\tfrac{1}{2}\right) = \left(\frac{a+c}{2}, \frac{b+d}{2}\right).$$

7. Choosing an appropriate coordinate system, we may assume one of the points of the parallelogram is the oringin in $\mathbb{R}^2$.

Denote the other three points by $P = (a,b)$, $Q = (c,d)$, $R = (a+c, b+d)$.

According to Prob. 6,

the midpoint of the line segment joining the points (a,b) and (c,d) is $\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$, and that of the line segment joinng the points $(0,0)$ and $(a+c, b+d)$ is $\left(\frac{0+(a+c)}{2}, \frac{0+(b+d)}{2}\right) = \left(\frac{a+c}{2}, \frac{b+d}{2}\right)$

So, the two diagonals meet in their respective midpoint.