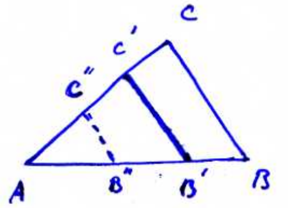
HW 13 - 1.

7.5;



Similar to proof of Lemma 7.2, let B'' be a point on AB such that $n|\overline{AB''}| = |AB|$.

Then, draw $\overline{B''C''}$ parallel to $\overline{BC}$.

By Lemma 7.2, $\frac{|AB''|}{|AB|} = \frac{|AC''|}{|AC|} = n$.

On the other hand, $\left\{ \begin{array}{l} \frac{|AB'|}{|AB|} = \cancel{} = \cancel{} \frac{n}{m} \\ \frac{|AB''|}{|AB} = \frac{|AC''|}{|AC|} = n \end{array} \right. \longrightarrow \frac{|AB''|}{|AB'|} = \cancel{} = \frac{1}{m}$

which by Ex. 7.4, and by the fact that $\overline{B''C''} \parallel \overline{BC} \parallel B'C'$ gives $\frac{|AB'|}{|AB|} = \frac{|AC'|}{|AC|} = \frac{n}{m}$.

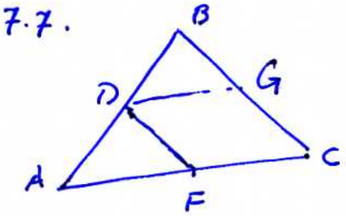
7.6. By construction, $|\overline{B'C''}| = |\overline{BC}|$. ①

$$\left.\begin{array}{l} \overline{BC} \parallel \overline{B'C'} \longrightarrow \left\{ \begin{array}{l} m\angle BCA = m\angle B'C'A \\ \angle ABC = m\angle AB'C' \ ② \end{array}\right. \\ \overline{A'C''} \parallel \overline{AC'} \longrightarrow m\angle B'C'A = m\angle B'C''A' \end{array}\right\} \longrightarrow$$

$m\angle B'C''A' = m\angle BCA$. ③

$$1,2,3 \xrightarrow{ASA} \triangle AB'C'' \cong \triangle ABC \quad \blacksquare$$

7.7.



(1) By Ex. 7.2, $DF \parallel BC$.

(2) ~~$\frac{|AD|}{|AB|}$~~ As proof of Lemma 7.2,

Draw $DG \parallel AC$. →

$$\frac{|BG|}{|BC|} = \frac{|BD|}{|BA|} = \frac{1}{2} \quad \left(\frac{|BD|}{|AB|} = 1 - \frac{|AD|}{|AB|} = 1 - \frac{1}{2} \right.$$

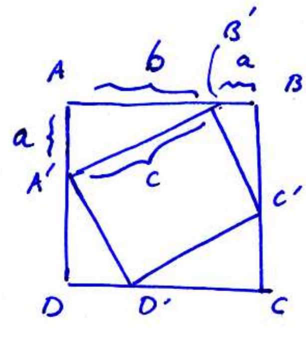
By construction $FD \parallel GC$
By part (1) $DG \parallel AC$ → $\Diamond FDGC$ is a parallelogram → $|CG| = |$

HW13 - 2 +

7.7 - 2 - Cont'd: $\bigtriangledown$ → I hope it is not a reserved symbol, for a religious sect, music band, or things like them!

$\bigtriangledown$ give us $|DF| = |BG| = |GC| \Rightarrow |DF| = \frac{1}{2}|BC|$

7.11) (1)-(2)



The area of the (allegedly!) square $\lozenge ABCD$ is

$(b+a)^2$

(Here, we're using the fact that the area of a square is length of a side squared.)

On the other hand, $\text{Area}(\lozenge ABCD) = \text{Area}(\overset{\triangle}{DC'C}) + \text{Area}(\overset{\triangle}{AA'B'}) + \text{Area}(\overset{\triangle}{DD'A}) + \text{Area}(\lozenge A'B'C'D') + \text{Area}(\overset{\triangle}{BB'C'})$ (*)

(Here, we're using the fact that area of two disjoint sets is the addition of their areas).

Rewriting (*)

$A(\lozenge ABCD) = A(\lozenge A'B'C'D') + 4A(\overset{\triangle}{AA'B'})$ **

(We used the fact that the area of two congruent triangles is equal

$** \longrightarrow (a+b)^2 = c^2 + 4 \times \frac{1}{2}ab$

$a^2+b^2+2ab = c^2 + 2ab \longrightarrow a^2+b^2=c^2$ ▨