IMPROPER INTEGRALS: ESCAPE VELOCITY

Kynetic Energy: of a mass m moving at speed v at some instant t_1, that is $v = v(t_1)$:

$$1/2\, m\, v^2.$$

Why?

Energy = Work

$$W = \int_{x_0}^{x_1} F\, dx$$

$\int_{x_0}^{x_1} F\, dx = \int_{x_0}^{x_1} m\, a\, dx =$ USE THE SUBSTITUTION RULE!!!
$\int_{t_0}^{t_1} m\, a\, dx/dt\, dt = \int_{t_0}^{t_1} m\, a\, v\, dt = \int_{t_0}^{t_1} m\, v'\, v\, dt.$

$\int_{t_0}^{t_1} m\, v'\, v\, dt =$ USE INTEGRATION BY PARTS!!!
$= 1/2\, m(v^2(t_1) - v^2(t_0) =$
$1/2\, m\, v^2(t_1) = 1/2\, m\, v^2$ if the initial velocity is zero, as it should since we are computing the work to make the particle move from speed zero to speed v.

What is the speed v_e needed to escape the gravitational pull of earth?

The gravitational force of earth on an object of mass m at distance r from the center of earth is:

$$F = F(r) = G\, M\, m/r^2,$$

$G =$ the gravitational constant, $M =$ earth's mass.

Starting from the earth surface, that is at distance $R =$ earth's radius from the center, neglecting frictions etc, you must have enough kynetic energy to win the pull which will decrease the kynetic energy in view of the fact that the gravitational force is working against your vertical escaping motion:

$$W = \int_R^{\infty} GMm/r^2\, dr = 1/2\, mv_e^2.$$

$$\lim_{b \to \infty} (-GMm/r\, |_R^b) = GMm/R = 1/2\, mv_e^2.$$

$$v_e = \sqrt{2GM/R}$$

Now check any physics book or encyclopedia and find: G, M and R and determine v_e.