Finite, infinite ... and counting

Finite sets.
Infinite sets.
Cardinality.
Bigger/smaller.
The integers.
Countability.
What is the cardinality of a finite union of finite sets?
What is the cardinality of a countable union of finite sets?
What is the cardinality of the union of two countable sets?
What is the cardinality of the union of finitely many countable sets?
What is the cardinality of a countable union of countable sets?
The rational numbers.
The algebraic numbers.
The decimal representation of any real number.
Are there only countably many real numbers?
The diagonal counting trick.

Is there a non-countable set whose cardinality is smaller than the cardinality of the real numbers?