

MAT 142 Spring 2003 FINAL

!!! WRITE YOUR NAME, STUDENT ID BELOW !!!

NAME :

ID :

**THERE ARE 11 PROBLEMS. THEY DO NOT HAVE EQUAL VALUE.
SHOW YOUR WORK!!!**

1	40	
2	30	
3	30	
4	20	
5	30	
6	30	
7	40	
8	dropped	
9	30	
10	20	
11	30	
Total	300	

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1. Solve the following differential equation with initial value:

$$\frac{dy}{dx} = x + y, \quad y(0) = 0.$$

2. A semicircular plate 2 ft in diameter is submerged vertically into a fluid with the diameter along the surface. The weight-density of the fluid is 60 lb/ft^3 .

Find the force exerted by the fluid on one side of the plate.

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3. Use the definition of the natural logarithm

$$\ln x = \int_1^x \frac{1}{t} dt, \quad x > 0,$$

to prove that, for $a > 0$:

$$\ln ax = \ln a + \ln x.$$

4. Integrate:

a)

$$\int \frac{1}{\sqrt{4x + x^2}} dx.$$

b)

$$\int \cos(\ln t) dt.$$

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5.

a) Find the Taylor series generated by $f(x) = x^5$ at $x = 1$.

b) Use a), or any other method, to write the following expression using partial fractions:

$$y = \frac{x^5}{(x-1)^6}.$$

6. Find a value c that makes the function $f(x)$ continuous at $x = 0$, where

$$f(x) = \frac{\sin(-7x)}{\tan(11x)}, \quad x \neq 0, \quad f(0) = c.$$

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7. Study the convergence of the following series.

a)

$$\sum_{n=10}^{\infty} \frac{(\ln n)^2}{n^4}$$

b)

$$\sum_{n=1}^{\infty} \frac{n!}{(2n)!}$$

c)

$$\sum_{n=1}^{\infty} (-1)^{n+1} \left(1 + \frac{1}{n}\right)$$

8. Find a sequence a_n such that all three conditions below are satisfied:

- 1) $\lim_{n \rightarrow \infty} a_n = 0$,
- 2) $a_n \geq 0$ for all n and
- 3) $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ diverges.

9. Study the following power series (radius of convergence, interval of convergence, convergence at the endpoints).

a)

$$\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}.$$

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b)

$$\sum_{n=2}^{\infty} \frac{(x-1)^n}{\sqrt{n} \ln n}.$$

10. Find the Maclaurin series of the following function

$$f(x) = x \ln(1 + x^2).$$

11. Prove, using (ϵ, N) , that

$$\lim_{n \rightarrow \infty} \left(-1 - \frac{1}{n^{\frac{2}{3}}}\right) = -1.$$