MAT 131 FALL 2012 MIDTERM I

NAME : ID :

RECITATION NUMBER:

THERE ARE SIX (6) PROBLEMS. THEY HAVE THE INDICATED VALUE.

SHOW YOUR WORK

DO NOT TEAR-OFF ANY PAGE

NO CALCULATORS NO CELLS ETC.

ON YOUR DESK: ONLY test, pen, pencil, eraser.

1		40pts
2		40pts
3		40pts
4		50pts
5		40pts
6		40pts
Total		250pts

!!! WRITE YOUR NAME, STUDENT ID AND LECTURE N. BELOW !!!

NAME : ID :

LECTURE N.

1. (40pts)

Find the domain and range of the function $y = \sqrt{x^2 - 1} - 1$.

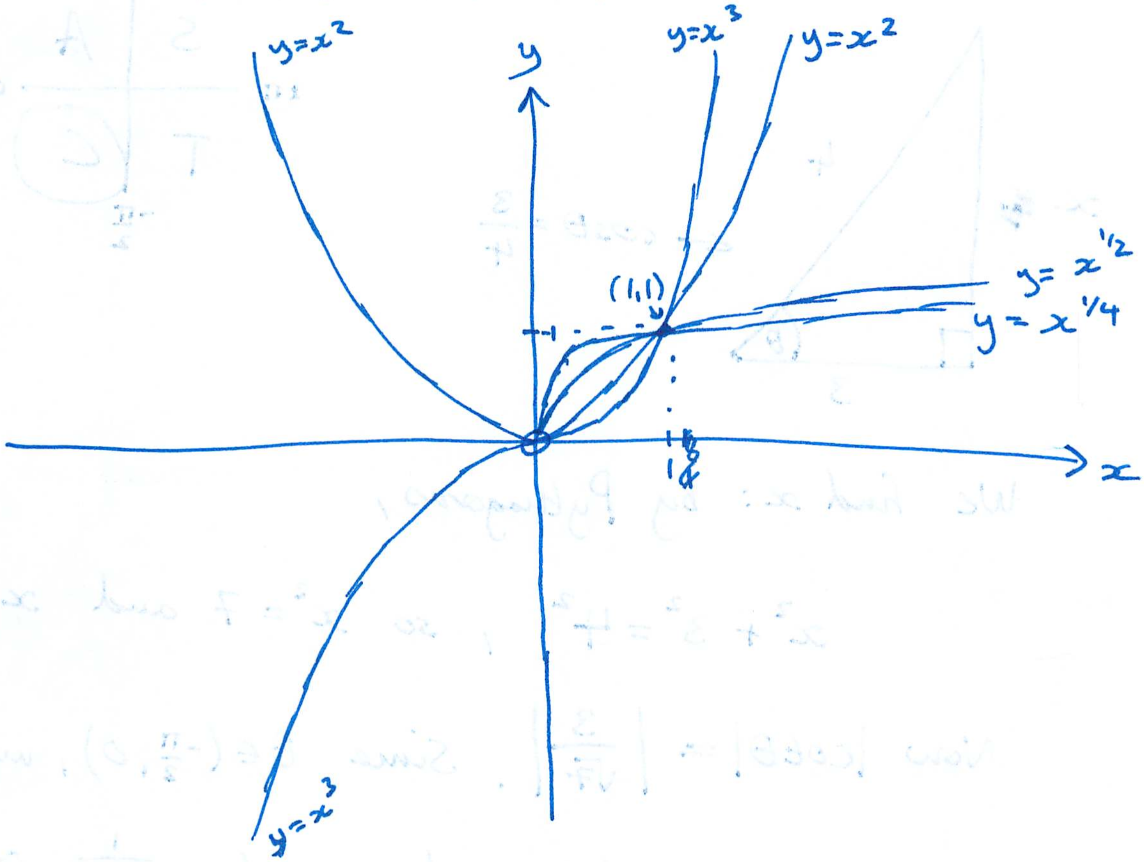
DOMAIN The function is defined whenever $(x^2-1) \geq 0$, since the square root function $g(x) = \sqrt{x}$ has domain $[0, \infty)$. Since $x^2 \geq 1$ whenever $x \leq -1$ or $x \geq 1$, the domain of the function is $\boxed{D = (-\infty, -1] \cup [1, \infty)}$.

RANGE The function $g(x) = \sqrt{x}$ takes only values greater than or equal to 0. It follows that the function $y = \sqrt{x^2-1} - 1$ takes values greater than or equal to -1. Hence the ~~domain~~ range of the function is $\boxed{R = [-1, \infty)}$.

2. (40pts)

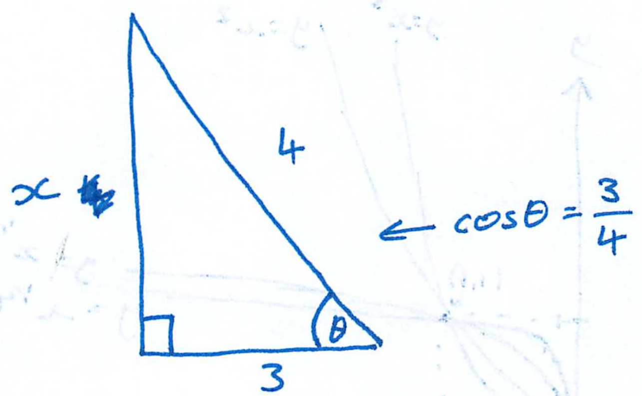
Graph on the same xy-plane the four functions by indicating the points where they intersect each other and the axes:

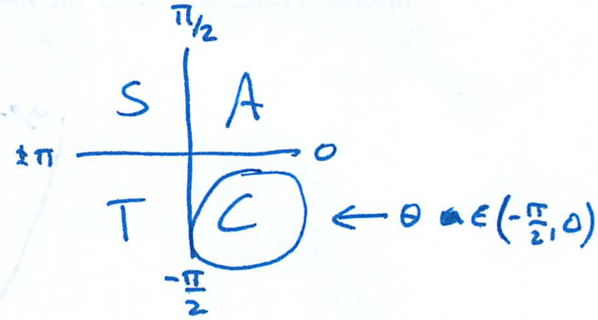
$$y = x^2, \quad y = x^{1/2}, \quad y = x^3, \quad y = x^{1/4}.$$



3. **(40pts)**

If $\cos\theta = 3/4$ and $-\pi/2 < \theta < 0$, find $\cot\theta$.





We find x: by Pythagoras,

$x^2 + 3^2 = 4^2$, so $x^2 = 7$ and $x = \sqrt{7}$.

Now $|\cot\theta| = \left|\frac{3}{\sqrt{7}}\right|$. Since $\theta \in (-\frac{\pi}{2}, 0)$, we know that $\tan\theta$ is negative, and so $\cot = \frac{1}{\tan\theta}$ is also negative. So

$$\boxed{\cot\theta = -\frac{3}{\sqrt{7}}}$$

4. (50pts)

An isotope of a certain element has a half-life of 9 hours. A sample of this isotope has a mass of 30 grams.

(a) How much of the sample remains after 18 hours? How much remains after 45 hours?

After 18 hours: $30 \times \frac{1}{4} = 7.5g$

After 45 hours: $30 \times \frac{1}{2^5} = \frac{15}{16} g.$

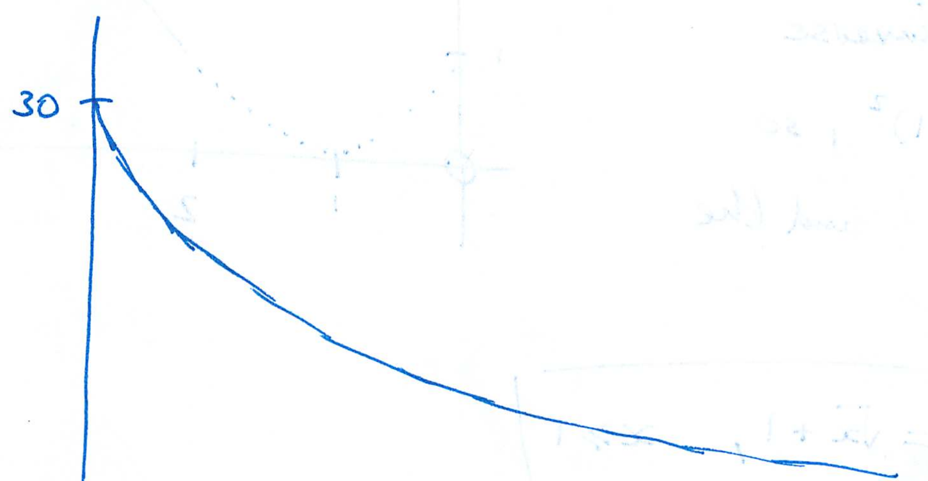
(b) Find a function which models the mass $m(t)$ of the sample after t hours. Give the domain and range of this function.

The function is

$$\boxed{m(t) = 30 \times \left(\frac{1}{2}\right)^{t/9}}$$

$\boxed{\text{Domain is } \{t \mid t \geq 0\}. \quad \text{Range is } (0, 30].}$

(c) Sketch the graph of the function you found in the previous part of the question.



5. (40pts)

(a) State the *horizontal line test.*

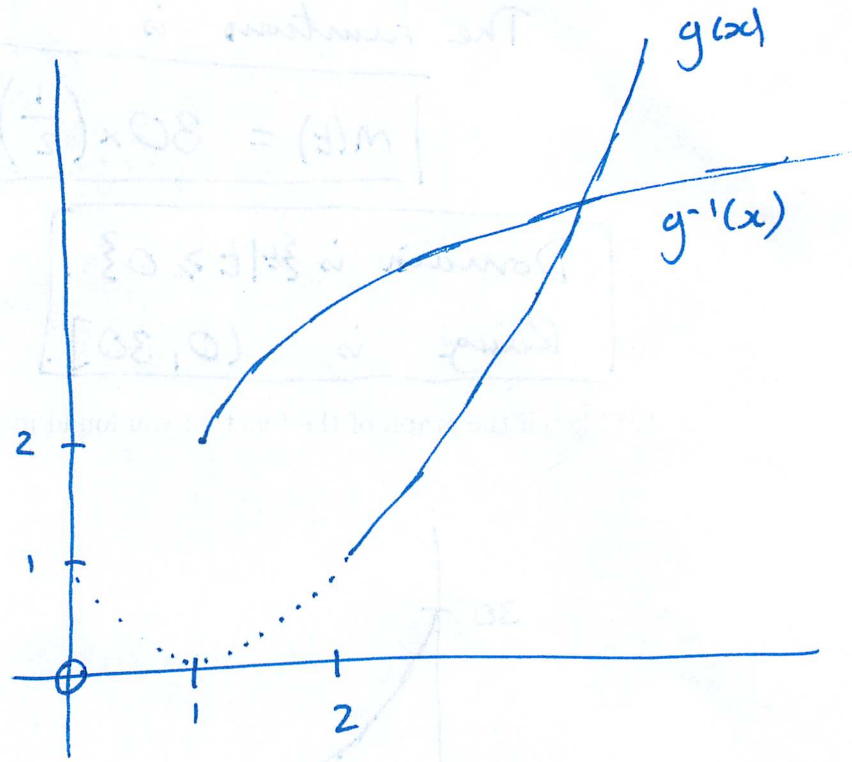
A Function is one-to-one if and only if no horizontal line intersects its graph more than once.

(b) Sketch the graphs of the following function and (using the previous part or otherwise) state whether it has an inverse:

$$g(x) = (x-1)^2, \quad x \geq 2.$$

If it does, then calculate the inverse and sketch it on the same graph as the original function.

g passes the horizontal line test and is ~~has~~ ~~an inverse~~ a one-to-one function, and thus has an inverse.



Find the inverse

$y = (x-1)^2$, so

$\sqrt{y} + 1 = x$ and the

inverse is

$$\boxed{g^{-1}(x) = \sqrt{x} + 1, \quad x \geq 1}$$

(since g is defined on $x \geq 2$, we see that the inverse must be defined on the range of g which is $[1, \infty)$)

6. (40pts)

The following table gives the distance (in meters) covered by an athlete in the first few seconds of a race.

Time (s)	0	0.5	1	1.5	2	2.5	3
Distance (m)	0	2.1	5.2	8.0	12.1	16.4	20.8

(a) Calculate his average speed between:

- 1 second and 1.5 seconds

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{8-5.2}{0.5} = \boxed{5.6\,m/s}$$

- 1.5 seconds and 2 seconds

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{12.1-8.0}{0.5} = \boxed{8.2\,m/s}$$

(b) Give an estimate of the instantaneous speed of the runner after 1.5 seconds of the race. Explain how you obtained your answer and why the estimate is justified.

The runner is accelerating throughout, so it seems ~~so~~ that his speed at 1.5s should lie between 5.6m/s and 8.2m/s. So we take the average and guess the speed is

$$\frac{1}{2}(8.2 + 5.6) = \boxed{6.9\,m/s}$$