This is the only problem I was emailed about. It involves the homework portion of proving theorem 5.3 (SSS). For completeness, I will restate the theorem and then proceed to prove the entire bit. This will be difficult to follow without pictures as a guide, so please try to draw the pictures for yourself while reading through this.

Theorem 5.3 (SSS) If $\triangle ABC$ and $\triangle A'B'C'$ are such that $|AB| = |A'B'|$, $|AC| = |A'C'|$, and $|BC| = |B'C'|$, then $\triangle ABC \cong \triangle A'B'C'$.

Proof. If the two triangles were not congruent, then one of the angles of $\triangle ABC$ would have to be different from the corresponding angle in $\triangle A'B'C'$. If necessary, relabel the triangles so that $m\angle A$ is different from $m\angle A'$, and also relabel so that $m\angle A' < m\angle A$.

Choose a point D so that the ray $\overrightarrow{AD}$ is interior to $\angle A$, and so that $m\angle DAB = m\angle A'$, and also so that $|AD| = |A'C'|$. The protractor axiom, ruler axiom, and Exercise 4.4 allow us to do this. As of right now, we don't know where the point D is. The point D could lie on the line segment $\overline{BC}$, it could lie on the other side of the line $\overleftrightarrow{BC}$, or inside the triangle $\triangle ABC$. We have to approach each case separately. Note that the first and last cases correspond to A and D being on "opposite sides" or the "same side" of the line $\overleftrightarrow{BC}$, respectively. Note that in all three cases, however, that B and C are always on "opposite sides" of the line $\overleftrightarrow{AD}$. *Why?*

Case 1: Suppose D lies on the line segment $\overline{BC}$. This is the easiest case. Since $m\angle DAB = m\angle A'$, $|AB| = |A'B'|$, and $|AD| = |A'C'|$, the Side-Angle-Side Axiom tells us that the triangles $\triangle A'B'C'$ and $\triangle ABD$ are congruent. If $\triangle A'B'C' \cong \triangle ABD$, by the definition of congruence, we must have $|B'C'| = |BD|$. But since D is in the line segment $\overline{BC}$, we must also have $|BD| < |BC|$. But our assumption was SSS, so we also have $|BC| = |B'C'| = |BD| < |BC|$, which is a contradiction.

Case 2: Suppose next that D is outside the triangle $\triangle ABC$, or equivalently, that A and D are on opposite sides of the line $\overleftrightarrow{BC}$. In this case we can consider the line segments $\overline{BD}$, $\overline{CD}$, and $\overline{AD}$. By construction we have $m\angle DAB = m\angle A'$ and $|AD| = |A'C'|$; and by our initial assumption $|AB| = |A'B'|$. Thus by the Side-Angle-Side Axiom, we have $\triangle ABD \cong \triangle A'B'C'$. Since they're congruent, we have $|BD| = |B'C'|$; and by our initial assumption, we have $|BC| = |B'C'|$. Thus we get $|BD| = |B'C'| = |BC|$. So now, we see that the triangle $\triangle BDC$ is isosceles with base $\overline{CD}$. By construction, $|AD| = |A'C'|$ and by our initial assumption, $|AC| = |A'C'|$. Thus we have another isosceles triangle $\triangle ADC$ also with base $\overline{CD}$.

Since the line segment $\overline{AD}$ intersects the line segment $\overline{BC}$, we may use Crossbar Theorem to say that the ray $\overrightarrow{DA}$ is interior to the angle $\angle BDC$. By definition, this means that the angle $\angle ADC$ is an interior angle of $\angle BDC$. So by the Monotonicity of Angles Theorem, we must have $\angle ADC < \angle BDC$. We may use the same argument again to say that since the line segment $\overline{CB}$ intersects the line segment $\overline{AD}$, the angle $\angle BCD$ is interior to the angle $\angle ACD$. So again by the Monotonicity of Angles Theorem, we have $m\angle BCD < m\angle ACD$.

Since the triangles $\triangle ADC$ and $\triangle BDC$ are isosceles, both with base $\overline{CD}$, we may use the Base Angles Equal Theorem to say that $m\angle BCD = m\angle BDC$ and $m\angle ACD = m\angle ADC$. Combining these equalities and inequalities, we get

$$m\angle ADC < m\angle BDC = m\angle BCD < m\angle ACD = m\angle ADC$$

Thus we have $m\angle ADC < m\angle ADC$, which is a contradiction.

Case 3: Suppose finally that D is inside the triangle $\triangle ABC$, or equivalently, that A and D are on the same side of the line $\overleftrightarrow{BC}$. In this case we can again consider the line segments $\overline{BD}$, $\overline{CD}$, and $\overline{AD}$. The same arguments from the beginning of *Case 2* still apply to this case. *Why?* So we may again say that the $\triangle ABD \cong \triangle A'B'C'$, and that the two triangles $\triangle ADC$ and $\triangle BDC$ are isosceles, both with base $\overline{CD}$.

Now, let E be a point on the line $\overleftrightarrow{BC}$ so that C is between B and E. Then by the Protractor Axiom, we must have $m\angle BCD + m\angle ACD + m\angle ACE = \pi$. It follows that $m\angle BCD + m\angle ACD < \pi$ since $m\angle ACE > 0$.

Next, choose a point F on the line $\overleftrightarrow{AD}$ so that D is between A and F. If we can prove that the ray $\overrightarrow{DF}$ is interior to the angle $\angle BDC$, we can use a trick similar to the previous one. Since our initial construction put the ray $\overrightarrow{AD}$ interior to the angle $\angle CAB$, by the Crossbar Theorem, it intersects the line segment $\overline{BC}$. But since D is between A and F and since A and D are on the same side of the line $\overleftrightarrow{BC}$, the ray $\overrightarrow{DF}$ must also intersect the line segment $\overline{BC}$. So by the Crossbar Theorem, we have the ray $\overrightarrow{DF}$ is interior to the angle $\angle BDC$.

By the Protractor Axiom, we know that $m\angle ADC + m\angle CDF = \pi$. Since angle $\angle CDF$ is interior to angle $\angle CDB$, by the Monotonicity of Angles Theorem, we have $m\angle CDF < m\angle CDB$. Thus $m\angle ADC + m\angle CDB > \pi$. Now we remember that triangles $\triangle ADC$ and $\triangle BDC$ are isosceles, both with base $\overline{CD}$, so by the Base Angles Equal Theorem we have $m\angle ADC = m\angle ACD$ and $m\angle CDB = m\angle BCD$. Now, combining our equalities and inequalities, we have

$$\pi < m\angle ADC + m\angle CDB = m\angle ACD + m\angle BCD < \pi$$

Thus we have $\pi < \pi$ which is a contradiction.