FINAL EXAM (PRACTICE)

Math 203 - Calculus III
July 9th, 2009

Name:

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Calculators are not allowed, so justify all of your answers algebraically to ensure full credit. Final answers do not have to be simplified completely, i.e. it is ok to have a final answer of the form $8^{2/3}$.
- Circle or otherwise indicate your final answers.
- This test has 10 problems and is worth 40 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

GRADING:

Problem	1	2	3	4	5	6	7	8	9	10	**Bonus**	**Total**
Score												
Out of											5	40

Problem 1.

(1) Find the equation of the plane passing through the points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.
(2) What is the angle between the plane above and the plane $x - z = 0$?
(3) Find an equation of the line of intersection between the two above planes.

Problem 2. Consider the curve $\alpha(t) = (t^2, 2t, \ln t)$.

(1) Find the length of α from $t = 1$ to $t = e$.
(2) Find the curvature of α at $t = 1$.

Problem 3. Compute the following limits, or explain why they do not exist.

(1) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2}{x^2 + y^2}$

(2) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$

(3) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - y^2}{x - y}$

Problem 4. Find the equation of the tangent plane to the surface $f(x, y) = x^2 \arctan(xy)$ at the point $(1, 1)$.

Problem 5. Use the chain rule to compute $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ at $(1, 1)$ where

$$z = xe^{y+z}$$

and

$$x = u^2$$
$$y = \ln(u + v)$$
$$z = e^{uv}$$

Problem 6. In what direction does $f(x, y, z) = xye^{\sin z}$ increase most rapidly at the point $(1, 2, \pi/2)$? (it is not necessary to normalize your answer)

Problem 7. Find all local maxmima, minima and saddles of the function $f(x, y) = x^4 + y^4 - 4xy + 2$.

Problem 8. Find the volume of the region in the first octant and under the surface $f(x, y) = 4 - x^2 - y^2$.

Problem 9. Set up the integral of the function $f(x, y, z) = e^{x^2+y^2+z^2}$ over the region enclosed by the surface $1 + x^2 + y^2$, the cylinder $x^2 + y^2 = 5$ and the xy-plane.

Problem 10. Evaluate the line integral $\int_\alpha F \cdot d\alpha$, where F is the vector field $F(x, y) = (e^y, 2xe^y)$ and α is the square with sides $x = 0, x = 1, y = 0$ and $y = 1$.