

# FINAL EXAM (PRACTICE)

Math 125 - Calculus A  
August 21st, 2008

Name: \_\_\_\_\_

**Read all of the following information before starting the exam:**

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Calculators are not allowed, so justify all of your answers algebraically to ensure full credit. Final answers do not have to be simplified completely, i.e. it is ok to have a final answer of the form  $8^{2/3}$ .
- Circle or otherwise indicate your final answers.
- This test has 20 problems and is worth 40 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

GRADING:

<b>Page</b>	2	3	4	5	6	7	<b>Bonus</b>	<b>Total</b>
<b>Score</b>								
<b>Out of</b>	8	7	4	6	7	8	4	40

**Problem 1** (2 points). Find the domain of the function

$$f(x) = \frac{1}{\sqrt{x^2 - 2x - 3}}$$

**Problem 2** (2 points). Sketch the graph of the following function

$$f(x) = 1 - \cos(2x)$$

**Problem 3** (1 point each). Find the exact value of

1.  $\log\left(\frac{e^2}{\sqrt{e}}\right)$

2.  $e^{\log 2}$

3.  $\sin \pi/3 + \cos \pi/4$

4.  $\tan^{-1} \sqrt{3}$

**Problem 4** (3 points). Sketch the graph of a function  $f(x)$  such that

- |                                               |                                                  |
|-----------------------------------------------|--------------------------------------------------|
| 1.) $\lim_{x \rightarrow 0^-} f(x) = 1$       | 2.) $\lim_{x \rightarrow 0^+} f(x) = 2$          |
| 3.) $f(0) = 0$                                | 4.) $\lim_{x \rightarrow 2} f(x) = \infty$       |
| 5.) $\lim_{x \rightarrow -2^-} f(x) = \infty$ | 6.) $\lim_{x \rightarrow -2^+} f(x) = -\infty$   |
| 7.) $\lim_{x \rightarrow \infty} f(x) = 1$    | 8.) $\lim_{x \rightarrow -\infty} f(x) = \infty$ |
| 9.) $f(2) = f(-2) = \text{undefined}$         |                                                  |

**Problem 5** (1 point each). Evaluate each of the following limits.

1.  $\lim_{x \rightarrow 2} \frac{x^4 - 16}{x - 2}$

2.  $\lim_{x \rightarrow 0^-} \left( \frac{1}{x} - \frac{1}{|x|} \right)$

3.  $\lim_{x \rightarrow 1} e^{x^2+x+1}$

4.  $\lim_{x \rightarrow \infty} \frac{(x^2 - 1)(x^2 + 1)}{2x^4 + x^3 - 2x^2 + 1}$

**Problem 6** (2 points). Find the derivative of  $f(x)$  using the definition of the derivative (that is, do not use any differentiation rules).

$$f(x) = \frac{2x + 1}{x + 3}$$

**Problem 7** (2 points). Use linear approximations to find the approximate value of  $\sqrt[3]{8.1}$ .

**Problem 8** (1 point each). Compute the derivative of the following functions (you can use any rules you want).

1.  $f(x) = (x^3 + 1)^{10}(2x^2 + x - 1)^{20}$

2.  $g(x) = e^{\cos x} + \sin(\tan(x))$

3.  $h(x) = \frac{\log x}{\sin x + 2}$

**Problem 9** (3 points). Implicitly differentiate the following function and solve for  $y'$ .

$$\sin(xy) = x^3 + y^3$$

**Problem 10** (3 points). Sketch the graph of a function  $f(x)$  such that:

1.  $f$  is increasing on  $(-2, 2)$  and decreasing on  $(-\infty, -2)$  and  $(2, \infty)$
2.  $f$  attains a local minimum at  $x = -2$  and an absolute maximum at  $x = 2$
3.  $f$  is concave down on  $(-\infty, -3)$  and  $(0, 3)$  and concave up on  $(-3, 0)$  and  $(3, \infty)$

**Problem 11** (4 points). For the function  $f(x) = \frac{x^2}{x^2-9}$  determine the following information:

1. Vertical and horizontal asymptotes
2. Intervals increasing and decreasing
3. Intervals of concave up and down
4. Local maxima and minima, and points of inflection
5. Sketch a graph of  $f$  from the above information

**Problem 12** (2 points each). Compute the following limits.

1.  $\lim_{x \rightarrow \infty} e^{-x} \log x$

2.  $\lim_{x \rightarrow \infty} \frac{e^x}{x^{10}}$

**Problem 13** (2 points). Find the point on the parabola  $y = x^2$  closest to the point  $(3, 0)$ .

**Problem 14** (2 points). Find the antiderivative of  $f(x) = x^2 + \sec^2 x - e^x$ .