

FINAL EXAM

Math 132 - Calculus II
August 8th, 2007

Name: _____

Read all of the following information before starting the exam:

- Show all work, clearly and in order, if you want to get full credit. I reserve the right to take off points if I cannot see how you arrived at your answer (even if your final answer is correct).
- Calculators are not allowed, so justify all of your answers algebraically to ensure full credit. Final answers do not have to be simplified completely, i.e. it is ok to have a final answer of the form $8^{2/3}$.
- For problems asking to find a limit or determine convergence of a series you must prove your answer. You should state which theorems you are using to draw your conclusions.
- Circle or otherwise indicate your final answers.
- This test has 20 problems and is worth 40 points, plus some extra credit at the end. It is your responsibility to make sure that you have all of the pages!
- Good luck!

Problem 1 Evaluate the following integrals (2 points each):

a.) $\int \frac{\cos(\pi/x)}{x^2} dx$

b.) $\int \frac{\sqrt{x^2 - 1}}{x^4} dx$ Hint: use the substitution $x = \sec \theta$

c.) $\int_0^1 x^2 e^x dx$

d.) $\int_2^3 \frac{1}{x^2 - 1} dx$

Problem 2 State why the following integral improper, and determine if it converges or diverges (2 points). If it converges, find its value.

$$\int_{-\infty}^{\infty} x^2 e^{-x^3} dx$$

Problem 3 Sketch the region bounded by the following curves, and find the area of the region (2 points).

$$y = |x| \quad y = x^2 - 2$$

Problem 4 Find the volume of the region bounded by the following curves when rotated about the line $y = 1$ (2 points):

$$y = x \quad y = \sqrt{x}$$

Problem 5 Find the length of the curve $x = t^2, y = 5 + 3t^2$ where $0 \leq t \leq 1$ (2 points).

Problem 6 A rectangular aquarium is 1 meter long, 1 meter wide and 5 meters deep, and is full of water. Find the work needed to pump half of the water out of the aquarium (2 points). Remember that the acceleration due to gravity is $9.8 m/s^2$ and the density of water is $\rho = 1000 kg/m^3$.

Problem 7 For what values of r does the function $y = e^{rx}$ satisfy the differential equation $2y'' + y' - y = 0$ (2 points).

Problem 8 Solve the initial value differential equation (2 points):

$$\frac{dP}{dt} = \sqrt{Pt} \quad P(1) = 2$$

Problem 9 Solve the differential equation (2 points):

$$y'' - 5y' + 6y = 0$$

Problem 10 Determine if the sequence below converges or diverges. If it converges, find its limit (2 points).

$$a_n = \arctan\left(\frac{n^2 + n + 1}{n^2 - n - 1}\right)$$

Problem 11 Determine if each series below converges or diverges. If it converges, find its sum (2 points each)

a.) $\sum_{n=0}^{\infty} \frac{3^n}{\pi^{2n}}$

b.) $\sum_{n=0}^{\infty} \frac{1}{(\sqrt{2})^n}$

Problem 12 Determine if each series below converges or diverges (2 points). You do **not** need to find the sum if it converges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^2}$$

Problem 13 Find the radius of convergence and interval convergence of the following series (2 points each):

a.) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

b.) $\sum_{n=1}^{\infty} \sqrt{n}x^n$

Problem 14 Find the exact value of $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ (2 points).

Problem 15 Find the indefinite integral $\int \frac{\sin x}{x} dx$ as a power series (2 points).