

## MAT 125 Midterm II Review Sheet

### 2.5: Limits Involving Infinity

- $\lim_{x \rightarrow a} f(x) = \infty$  means that we can make  $f(x)$  as large as we want by plugging in an  $x$  close to  $a$ . Similarly,  $\lim_{x \rightarrow a} f(x) = -\infty$  means that we can make  $f(x)$  as small as we want.
- Vertical asymptotes occurring at  $x = a$  correspond to one-sided limits  $x \rightarrow a^+$  or  $x \rightarrow a^-$  being equal to  $\infty$  or  $-\infty$ . That is, if either (or both) one-sided limit at  $a$  is  $\pm\infty$ , we have a vertical asymptote there.
- Remember that rational functions have vertical asymptotes at  $a$  if the denominator is equal to 0 and the numerator is not equal to 0 at  $a$ . Also, recall that  $\log(x)$  has a vertical asymptote at 0 and that the tangent, cotangent, secant, and cosecant functions have vertical asymptotes.
- We say that  $\lim_{x \rightarrow \infty} f(x) = a$  if we get closer and closer to  $a$  as we put larger and larger  $x$  into the function. Similarly, we say that  $\lim_{x \rightarrow -\infty} f(x) = a$  if we get closer and closer to  $a$  as we put smaller and smaller  $x$  into the function.
- If we have a horizontal asymptote at  $a$ , we will have  $\lim_{x \rightarrow -\infty} f(x) = a$  or  $\lim_{x \rightarrow \infty} f(x) = a$ .
- Recall that we can determine the horizontal asymptote of a rational function using the following rules. If the leading term of  $p$  is  $p_n x^n$  and the leading term of  $q$  is  $q_m x^m$ , then:
  - If  $n < m$ , the x-axis is a horizontal asymptote.
  - If  $n = m$ , the line  $y = \frac{p_n}{q_n}$  is a horizontal asymptote.
  - If  $n > m$ , the graph has no horizontal asymptote. However, if  $n = m + 1$ , then there is a slant asymptote. You find the slant asymptote by dividing  $p$  by  $q$  and ignoring the remainder. Slant asymptotes give us limits at  $\pm\infty$  of  $\pm\infty$ .
- Recall that  $e^x$  has a horizontal asymptote at 0 and  $\tan^{-1}(x)$  has a horizontal asymptote at  $\pm\frac{\pi}{2}$ .
- If  $f(x)$  gets larger and larger as we put in larger and larger  $x$ , we say  $\lim_{x \rightarrow \infty} f(x) = \infty$ . If  $f(x)$  gets smaller and smaller as we put in larger and larger  $x$ , we say  $\lim_{x \rightarrow \infty} f(x) = -\infty$ . Also, the reverse statements with  $-\infty$  are also true.

### 2.6: Tangents, Velocities, and Other Rates of Change

- The tangent line to the curve at  $(a, f(a))$  is the line with slope  $\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a}$  through the point, provided that limit exists. If it does not, we say the tangent line does not exist. An equivalent formula for this slope is  $\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$ . This is just the limit of the slope of the secant line between two points as the points get closer together.
- If we're thinking about  $f(x)$  as the function of an object's position over time, we refer to the slope of the tangent line at  $a$  as the instantaneous velocity of the object at  $a$ .

### 2.7: Derivatives

- We call  $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$  the derivative of  $f$  at  $a$  if the limit exists. The derivative is the slope of the tangent line at  $a$ .
- If we want to graph a function when we know the derivative at certain points, we can draw the tangent lines (lines with slopes matching the derivative at that point) and then trace in a graph that looks like the tangent line at that point.
- The derivative is the rate that the function is changing value at a given point.

### 2.8: The Derivative as a Function

- We can also think of the derivative as a function. The formula for the derivative will be  $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ . We can think of this as a function that takes  $x$  to the slope of the tangent line of  $f$  at  $x$ .

- We say that a function is differentiable at  $a$  if the derivative exists at that point (that is, the limit exists). We say that  $f$  is differentiable on an interval if it is differentiable at all points in that interval. In terms of the graph, these intervals will correspond to areas where the function is continuous and it has no “corners” or vertical tangent lines. A corner is a sharp turn in the graph, like at 0 on the graph of  $|x|$  or at 0 on  $|\sqrt{x}|$  (which also has a vertical tangent line at 0, and thus fails in two different ways to be differentiable).
- The second derivative is the derivative of the derivative:  $f''(x) = \lim_{h \rightarrow 0} \frac{f'(x+h) - f'(x)}{h}$ . If we’re thinking about  $f$  as a function of position, we call  $f''(x)$  the acceleration. Similarly, the  $n$ th derivative is the derivative of the  $(n - 1)$ th derivative.

## 2.9: Linear Approximations

- We can think of the tangent line at  $a$  as a line that looks a lot like the function itself near  $a$ . We can use this function to estimate the value of the function at a point near a point where we know the derivative.
- If we know the derivative of a function at  $a$  and the value of it at  $a$ , we can use this derivative as the slope and the value to find the line. Then, we can approximate values of the function by plugging the  $x$  value into the tangent line equation. These values will be either overestimates or underestimates depending on whether the tangent line is above or below the function.

## 2.10: What does $f'$ say about $f$ ?

- If  $f'(x)$  is greater than 0,  $f$  is increasing at  $x$ . If  $f'(x) < 0$ , then  $f$  is decreasing at  $x$ . If  $f'(x) = 0$ , we have a horizontal tangent line, so  $f$  has a local maximum, local minimum, or point of inflection at  $x$  (depending on what the second derivative is at  $x$ ).
- If  $f''(x) > 0$ ,  $f$  is concave up at  $x$  (this means it opens up, like  $x^2$ ). If  $f''(x) < 0$ ,  $f$  is concave down at  $x$  (this means it opens down, like  $-x^2$ ). If  $f''(x) = 0$ , we have an inflection point, which is a point where concavity changes.
- The antiderivative is just the function that has  $f$  as a derivative. We can determine its graph the same way that we determined the graph of  $f$  from the graph of the derivative.

## 3.1: Derivatives of Polynomials and Exponential Functions

- The derivative of a constant function  $f(x) = c$  is  $f'(x) = 0$ .
- The derivative of  $f(x) = x$  is  $f'(x) = 1$ .
- If  $n$  is any number, then the derivative of  $f(x) = x^n$  is  $f'(x) = nx^{n-1}$ .
- If  $c$  is a constant, then the derivative of  $cf(x)$  is  $cf'(x)$ .
- If  $f$  and  $g$  are functions, then the derivative of  $f(x) + g(x)$  is  $f'(x) + g'(x)$ .
- If  $f$  and  $g$  are functions, then the derivative of  $f(x) - g(x)$  is  $f'(x) - g'(x)$ .
- The derivative of the exponential function,  $f(x) = e^x$ , where  $e \approx 2.71828$ , is  $f'(x) = e^x$ .

## 3.2: The Product and Quotient Rules

- The derivative of  $f(x)g(x)$  is equal to  $f'(x)g(x) + f(x)g'(x)$ .
- The derivative of  $\frac{f(x)}{g(x)}$  is equal to  $\frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$ .

## 3.4: Derivatives of Trigonometric Functions

- The derivative of  $\sin(x)$  is  $\cos(x)$ .
- The derivative of  $\cos(x)$  is  $-\sin(x)$ .
- The derivative of  $\tan(x)$  is  $\sec^2(x)$ .
- The derivative of  $\csc(x)$  is  $-\csc(x)\cot(x)$ .
- The derivative of  $\sec(x)$  is  $\sec(x)\tan(x)$ .
- The derivative of  $\cot(x)$  is  $-\csc^2(x)$ .