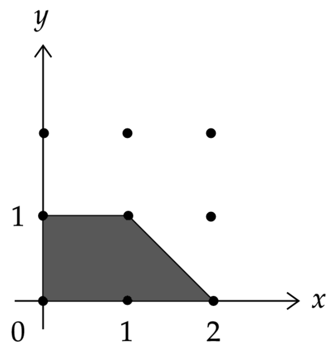
Homework 5 (MAT 562)

(1) Let (M, ω) be a symplectic toric manifold with moment map $\mu : M \longrightarrow \mathbb{R}^n$. Show that for each point p in the interior of the image of μ, we have that $\mu^{-1}(p)$ is a Lagrangian torus.

(2) Consider the symplectic toric manifold whose moment map image is the polytope below.



Write this symplectic toric manifold as an explicit moment map quotient of $\mathbb{C}^4$.

(3) State and prove the h-principle for contact structures on connected open manifolds.

(4) Using a parameterized h-principle for immersions, show that the following two torus embeddings are isotopic through immersions:

$$\iota_0 : S^1 \times S^1 \to \mathbb{R}^3, \quad \iota_0(e^{i\theta}, e^{i\phi}) = ((2 + \cos(\phi))\cos(\theta), (2 + \cos(\phi))\sin(\theta), \sin(\phi))$$
$$\iota_1 : S^1 \times S^1 \to \mathbb{R}^3, \quad \iota_1(e^{i\theta}, e^{i\phi}) = \iota_0(e^{i\theta}, e^{-i\phi}).$$

(5) Use an h-principle to construct a symplectic form on $\mathbb{R}^4$ with the property that it admits a Lagrangian submanifold diffeomorphic to S^2.