MAT 542 Homework 3

(1) Let $p_i : E_i \longrightarrow B$ be fibrations for $i = 0, 1$. Show that a fiber preserving map $f : E_0 \longrightarrow E_1$ is a fiber preserving homotopy equivalence iff it is a homotopy equivalence.

(2) (Hatcher p.419 Ex 1): Show that there is a map $\mathbb{R}P^\infty \longrightarrow \mathbb{C}P^\infty = K(\mathbb{Z}/2\mathbb{Z}, 2)$ which induces the trivial map on $\widetilde{H}(-, \mathbb{Z})$ but a non-trivial map on $\widetilde{H}^*(-, \mathbb{Z})$. How is this consistent with the universal coefficient theorem?

(3) (Hatcher p.419 Ex 1): Let G, H be groups and $K(G, n)$, $K(H, n)$ be CW complexes. Show that the map

$$\langle K(G,n), K(H,n)\rangle \longrightarrow \mathrm{Hom}(G,H), \quad [f] \longrightarrow (f_* : \pi_n(K(G,n)) \to \pi_n(K(H,n))$$

is an isomorphism.

(4) (Hatcher p.419 Ex 8): Show that $p : E \longrightarrow B$ is a fibration iff the map

$$\pi : E^I \longrightarrow E_p, \quad \pi(\gamma) := (\gamma(0), p \circ \gamma)$$

admits a section where E_p is the mapping path space construction associated to p.

(5) (Hatcher p.419 Ex 9): Let Δ be a 2-simplex and let $L : \Delta \longrightarrow I$ be a linear projection onto one of its edges. Show that L is a fibration but not a fiber bundle. (Hint: use previous exercise).