

- (1) Let $K = (\sigma_i)_{i \in I}$ be a simplicial complex. Show that the quotient topology on $|K|$ coming from the map

$$\sqcup_{i \in I} \sigma_i \rightarrow |K|$$

is identical to the usual topology on $|K|$.

- (2) For any subset J , show that addition

$$E^J \times E^J \rightarrow E^J, \quad (x, y) \rightarrow x + y$$

and scalar multiplication

$$\mathbb{R} \times E^J \rightarrow E^J, \quad (\lambda, x) \rightarrow \lambda x$$

are continuous functions.

- (3) Let

$$S^n := \{(x_0, \dots, x_n) \in \mathbb{R}^{n+1} : \sum_{j=0}^n x_j^2 = 1\} \subset \mathbb{R}^{n+1}$$

be the unit sphere. Let $\mathbb{R}P^n := S^n / \sim$ where \sim is the equivalence relation identifying x with $-x$. Is it true that $\mathbb{R}P^n$ is homeomorphic to a simplicial complex?

- (4) (Munkres Ch 1 ex 4,7)

- Let $\mathcal{S} \subset \mathcal{P}(\mathbb{N})$ be the abstract simplicial complex consisting of subsets of size ≤ 2 and where each subset of size 2 contains 0. Draw a geometric realization of such a simplicial complex and show its topology is not first countable.

- (5) (Munkres Ch 1 ex 7) Show each locally finite simplicial complex is metrizable (*Hint*: use barycentric coordinates).

- (6) Show that there is a finite simplicial complex in \mathbb{R}^n so that the union of its simplices in \mathbb{R}^n is the cube $[0, 1]^n \subset \mathbb{R}^n$.