Calculus IV, MAT 303
Summer II 2008
Practice final

(1) Solve the linear first-order equation

$$(x^2+1)y' + 3xy = 6x$$

Solution: Multiplication by the integrating factor

$$\rho(x) = e^{\int \frac{3x}{x^2+1}dx} = (x^2+1)^{3/2}$$

yields

$$(x^2+1)^{3/2}y = \int 6x(x^2+1)^{1/2}dx = 2(x^2+1)^{3/2} + C$$

(2) Find the general solution of the following linear equation

$$y^{(3)} + 3y'' + 4y' + 12y = 0$$

Solution: Solving the characteristic equation

$$r^3 + 3r^2 + 4r + 12r = 0$$

to get

$$r_1 = -3, \quad r_2 = 2i, \quad r_3 = -2i$$

hence the general solution is

$$y(x) = c_1 e^{-3x} + c_2 \cos 2x + c_3 \sin 2x.$$

(3) Solve the following linear system

$$\begin{aligned} x_1' &= 3x_1 + 2x_2 + 2x_3 \\ x_2' &= -5x_1 - 4x_2 - 2x_3 \\ x_3' &= 5x_1 + 5x_2 + 3x_3 \end{aligned}$$

Solution: First find the eigenvalue of the matrix, which is

$-2, 1, 3$ respectively, then

$$\begin{aligned} x_1(t) &= c_2e^t + c_3e^{3t} \\ x_2(t) &= c_1e^{-2t} - c_2e^t - c_3e^{3t} \\ x_3(t) &= -c_1e^{-2t} + c_3e^{3t} \end{aligned}$$

(4) Solve the Initial Valute Problem

$$X' = \begin{bmatrix} -2 & 1 \\ -1 & -4 \end{bmatrix} X, \quad X(0) = [0, 1]^t$$

Solution: The general solution is $x_1(t) = (c_1+c_2+c_2t)e^{-3t}, \quad x_2(t) = (-c_1-c_2t)e^{-3t}$, together with initial condition, $x_1(t) = te^{-3t}, \quad x_2(t) = (1-t)e^{-3t}$

(5) Use Laplace transformation to solve

$$x'' + x = \cos 3t, \quad x(0) = 1, \quad x'(0) = 0$$

Solution: $x(t) = \frac{1}{8}(9\cos t - \cos 3t)$

(6) Find the Laplace inverse transform of

$$F(s) = \frac{s}{(s-3)(s^2+1)}$$

Solution:

$$x(t) = \frac{1}{10}(3e^{3t} - 3\cos t + \sin t)$$