

MAT 312 Applied Algebra
Summer II 2009
Practice final

- (1) Find the last digit of $9^{3^{12}}$.
- (2) Find the sign of permutation $(123)(24)(134)(24)$
- (3) Find the smallest positive integer whose remainder when divided by 100 is 5, which is divisible by 15.
- (4) Let X and Y be two sets, show that $(X \cup Y)^c = X^c \cap Y^c$.
- (5) Let $f : X \rightarrow Y$, $g : Y \rightarrow X$ be two functions, show that if $f \circ g = id_Y$, then g is injective.
- (6) Let S_n be the n -th symmetric group, define a relation R on S_n by
 $(f, g) \in R$ if and only if $f = hgh^{-1}$ for some $h \in S_n$
show that R is an equivalence relation.
- (7) Let G be the set of all 2×2 invertible diagonal matrices with real entries; that is, matrices of the form

$$\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$$

Show that G is a group under matrix multiplication.

- (8) Let n be any integer greater than 1 and let a be relatively prime to n , then $a^{\phi(n)} \equiv 1 \pmod{n}$. (Euler's theorem)