MAT 126 Calculus B Fall 2005
Midterm II Answers

1. Evaluate the following indefinite integrals.

(a) (5 points)

$$\int \frac{x}{\sqrt{x^2+4}}\,dx = \sqrt{x^2+4} + C$$

by using the substitution rule with $u = x^2 + 4$.

(b) (5 points)

$$\int x^3(1+x^4)^{19}\,dx = \frac{(1+x^4)^{20}}{80} + C$$

by using the substitution rule with $u = 1 + x^4$.

(c) (10 points)

$$\int x^2 \ln x\,dx = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

by using integration by parts with $u = \ln x$, $dv = x^2\,dx$.

2. Evaluate the following definite integrals.

(a) (10 points)

$$\int_e^{e^4} \frac{\sqrt{\ln x}}{x}\,dx = \int_1^4 \sqrt{u}\,du = \frac{14}{3}$$

by using the substitution rule with $u = \ln x$.

(b) (10 points)

$$\int_0^1 xe^{2x}\,dx = \frac{1+e^2}{4}$$

by using integration by parts with $u = x$, $dv = e^{2x}\,dx$.

3. Evaluate the following indefinite integrals.

(a) (10 points)

$$\int \cos^5 t\,dt = \sin t - \frac{2\sin^3 t}{3} + \frac{\sin^5 t}{5} + C$$

by using the fundamental trigonometric identity and the substitution rule with $u = \sin t$.

(b) (10 points)

$$\int \frac{x^2-1}{x^2+1}\,dx = \int \left(1 - \frac{2}{x^2+1}\right) dx = x - 2\tan^{-1} x + C$$

by using the long division.

(c) (5 points)

$$\int e^{\cos x} \sin x \, dx = -e^{\cos x} + C$$

by using the substitution rule with $u = \cos x$.

(d) (5 points)

$$\int \cot x \, dx = \int \frac{\cos x}{\sin x} \, dx = \ln|\sin x| + C.$$

4. (a) (5 points) Find the numbers A and B such that

$$\frac{x+6}{x^2 - 3x - 4} = \frac{A}{x-4} + \frac{B}{x+1}.$$

$A = 2$ and $B = -1$.

(b) (10 points) Evaluate the integral

$$\int \frac{x+6}{x^2 - 3x - 4} \, dx = 2\ln|x-4| - \ln|x+1| + C.$$

5. Evaluate the following definite integrals

(a) (5 points)

$$\int_{-\pi}^{\pi} \frac{\sin^3 x}{x^4 + 1} \, dx = 0$$

— integral of the odd function over the symmetric interval.

(b) (10 points)

$$\int_{-\pi}^{\pi} |\sin x| \, dx = 2\int_0^{\pi} \sin x \, dx = 4$$

— interval of even function over the symmetric interval.

Extra Credit (10 points) Evaluate the integral

$$\int_{\frac{\pi}{4}}^{\tan^{-1} e} \sec^2 \theta \ln(\tan\theta) d\theta = \int_1^e \ln u \, du = (u \ln u - u)\Big|_1^e = 1$$

by using the substitution rule with $u = \tan\theta$.