

MAT 126 Calculus B Fall 2005 Practice Final Exam

Answer each question in the space provided and on the reverse side of the sheets. Show your work whenever possible. Unless otherwise indicated, **answers without justification will get little or no partial credit!** Cross out anything that grader should ignore and circle or box the final answer. You **do not need** to simplify numerical answers or write their approximate values. This practice exam contains more problems than the actual test to give you more practice.

1. Evaluate the following definite integrals:

(a)

$$\int_1^9 \ln \sqrt{x} \, dx$$

(b)

$$\int_0^2 \frac{x}{1+2x^2} \, dx$$

(c)

$$\int_1^e \frac{(\ln x)^3}{x} \, dx$$

(d)

$$\int_{-1}^1 x^2 \sin(x^5) \, dx$$

(e)

$$\int_0^{1/2} \frac{\sin^{-1} x}{\sqrt{1-x^2}} \, dx$$

(f)

$$\int_1^4 \sqrt{t} \ln t \, dt$$

(g)

$$\int_0^{13} \frac{dx}{\sqrt[3]{(1+2x)^2}}$$

(h)

$$\int_0^{\pi/2} \sin^3 x \, dx$$

2. Evaluate the following indefinite integrals:

(a)

$$\int x^2 e^x \, dx$$

(b)
$$\int \frac{2x^3 + 1}{x^2 + 1} dx$$

(c)
$$\int \frac{\tan^{-1} x}{1 + x^2} dx$$

(d)
$$\int \sin^{-1} x dx$$

(e)
$$\int \frac{x - 1}{x^2 + 3x + 2} dx$$

(f)
$$\int t^2 \cos(1 - t^3) dt$$

(g)
$$\int e^x \sqrt[3]{1 + e^x} dx$$

(h)
$$\int \cos^5 x dx$$

3. (a) Write a formula for $\cos^2 x$ in terms of $\sin^2 x$.
 (b) Evaluate

$$\int \cos^3 x \sin^2 x dx$$

4. Let

$$f(x) = \int_2^{\sqrt{x}} \frac{\sin t}{t} dt + x^2$$

- (a) Find $f'(x)$.
 (b) Evaluate $f(4)$.
 5. Find a function f and a number a such that for x ,

$$1 + \int_a^x tf(t) dt = x^3$$

6. (a) Let

$$I = \int_0^4 e^{x^2} dx$$

For any value of n list the numbers L_n, R_n, M_n, T_n and I in increasing order.

(b) Repeat part (a) for

$$I = \int_0^{\sqrt{2}/2} e^{-x^2} dx$$

7. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

(a)

$$\int_0^{\infty} e^{-x} dx$$

(b)

$$\int_0^1 \frac{1}{\sqrt{x}} dx$$

(c)

$$\int_0^3 \frac{1}{x\sqrt{x}} dx$$

(d)

$$\int_{-\infty}^{\infty} xe^{-x^2} dx$$

(e)

$$\int_0^1 \frac{1}{4y-1} dy$$

8. Find the area of the region bounded by the curves:

(a) $y = x^2$ and $y = x^4$.

(b) $x + y^2 = 2$ and $x + y = 0$.

9. (a) Find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2$ and $y^2 = x$ about the x -axis.

(b) Find the volume of the solid of revolution obtained by rotating the region bounded by $y = \sec x$, $y = 1$, $x = -1$ and $x = 1$ about the x -axis.

10. Find the length of the following curves:

(a) $y = x^{3/2}$, $0 \leq x \leq 2$.

(b)

$$y = \frac{x^2}{4} - \frac{\ln x}{2}, \quad 1 \leq x \leq 2$$

11. Find the average value f_{ave} of f on the given interval.

(a) $f(x) = x \sin(x^2)$ on $[0, \sqrt{\pi}]$.

(b) $f(x) = 4 - x^2$ on $[0, 3]$.

(c) For f as in part (b) find the number c in $[0, 3]$ such that $f(c) = f_{ave}$.