Section 7.2

4. $y' = y - x$

6. $y' = y^3 - x^3$

Compare the graph I and II.

$$\begin{aligned} y' &= y^3 - x^3 \\ &= (y-x)(y^2 + yx + x^2) \end{aligned}$$

$y^2 + yx + x^2 > 0$, so that $|y^3 - x^3| \geq |y - x|$ for all real number x, y

Hence, if we select the same point in the planes of I and II, then the direction of the vector which start at that given point in equation 6. is closer to the y-axis than that of equation 4.
In other words, if the same point of each plane is given, then the vectors of equation 6 are more upright than that of equation 4.

Therefore, the graph of the vector field of 4 is II, and the graph of the vector field of 6 is I

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22. $y' = 1 - xy, \qquad y(0) = 0$

Step size $h = 0.2$

Let $y' = F(x, y) = 1 - xy$

By Euler's method,

$$\begin{aligned} y(0.2) \approx y_1 &= y_0 + hF(x_0, y_0) \\ &= 0 + 0.2 \cdot 1 \\ &= 0.2 \end{aligned}$$

$x_0 = 0, \quad y_0 = 0,$

$F(x_0, y_0) = 1$

Apply this method recursively until we get the approximate value of $y(1.0)$.

$$y(0.4) \approx y_2 = y_1 + hF(x_1, y_1) = 0.2 + 0.2 \cdot 0.96 = 0.392$$

$$x_1 = 0.2, \quad y_1 = 0.2, \quad F(x_1, y_1) = 1 - 0.2 \cdot 0.2 = 0.96$$

$$y(0.6) \approx y_3 = y_2 + hF(x_2, y_2) = 0.392 + 0.2 \cdot 0.8432 = 0.56064$$

$$x_2 = 0.4, \quad y_2 = 0.392, \quad F(x_2, y_2) = 1 - 0.4 \cdot 0.392 = 0.8432$$

$$y(0.8) \approx y_4 = y_3 + hF(x_3, y_3) = 0.56064 + 0.2 \cdot 0.663616 = 0.6933232$$

$$x_3 = 0.6, \quad y_3 = 0.56064, \quad F(x_3, y_3) = 1 - 0.6 \cdot 0.56064 = 0.663616$$

$$y(1.0) \approx y_5 = y_4 + hF(x_4, y_4) = 0.6933632 + 0.2 \cdot 0.44530944 = 0.782425088$$

$$x_4 = 0.8, \quad y_4 = 0.6933632, \quad F(x_4, y_4) = 1 - 0.8 \cdot 0.6933632 = 0.44530944$$

Therefore, $y(1.0) \approx 0.782425088$