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Notes from class on 9/17/09

<u>Review of Homework #1</u>

Prove that irrational numbers are not closed under addition:
Proof: *2* is irrational
-*2* is irrational
2 + (- *2*) = 0

- This shows us that irrational numbers are not closed under addition because 0 is not irrational.
- We can also state that **Q** intersect **~Q** is equal to the empty set.

A student wrote:
Suppose e and e' denote 2 identity elements.
For every a within G
ae = ea = a
ae' = e'a = a
so, ae = ae' which we are really showing my multiplying either equation by a^{-1}
now we can say $aa^{-1} = e$ **OR** $aa^{-1} = e'$

We can take that information and deduce that: ee' = e and ee'=e'
However, the point of showing this proof is to see that it is NOT COMPLETE. Given this proof it is unclear whether it is e or e' which is our identity element. So while they had the right idea, the proof is not complete.

<u>Notes from 9/17/09</u>
The below example is to show us that integers can be viewed as similar to polynomials, because they are both rings.

Z → Polynomial ring w/ coefficients in a field
Integers

<u>Ring</u>(+, *) Integers

- Not all integers have multiplicative inverse
- 2 * ½ = 1, true, but ½ is not an integer

<u>Ring</u>(+, *) Polynomials

- The same can be said about polynomials, not all polynomials have multiplicative inverses that fall within our ring.

Question: We expect our high school students to understand the similarity between integers and polynomials, why is this expected?

Definition of a Polynomial:

An expression in the form: $a_n x^n + a_{n-1} x^{n-1} + \ldots\ldots + a_1 x^1 + a_0$

Provided that:

- $a_n \neq 0$
- n is an integer
- n is non-negative
    - note: the coefficients a_i lies in some ring.

Example: $X^3 - 5X^2 + 12X - 7$

We could take the coefficients of a polynomial to be rational functions.

1□2+1•X^3 + X^2 + *1*□ •X + *5*□*2*□*2* is a polynomial.

Rational Functions

First we look at integers,

Q = {□□: a,b € **Z** (b≠0) and □□ = □ □ if and only if ad=bc}

Now we can make a comparison to the set of rational functions

= { □*(*□*)*□*(*□*)*: p(x), q(x) in k[x], q(x)≠0 and □*1*□□*1*□ ~ □*2(*□*)*□*2(*□*)* if and only if $P_1(x)Q_2(x) = Q_1(x)P_2(x)$}

Note: k[x] is a polynomial ring where k is any field.

Now in the case with rational functions, what do we mean when we say that q(x)≠0????

Well, in words it means that the denominator of the quotient cannot be the zero polynomial, which is just 0. However, we can have a polynomial which given some root may give us zero. Such as:

□□*2−1* is a rational function. □*0* is **NOT** a rational function.

So in general □□□□ is a rational function as long as Q(x) *≠0* where 0 represents the 0 polynomial.

When we say polynomials are equivalent only if cross products are equal should we call them equal. Can we say: □*1*□□*1*□ = □*2(*□*)*□*2(*□*)* ????

Example: □*−1*□*2−1* ~ *1*□*+1*

One student said that: if we factor the first equation we can show □*−1*□*2−1*=□*−1*□*−1(*□*+1)* then we could cancel the (x-1) and be left with *1*□*+1*. This all may look like we should be able to use ~ and = interchangeably, but □*−1*□*2−1* and *1*□*+1* are not equal as functions. It is important to note that there are 2 separate ways of looking at these equations first we could look algebraically, or we could then look at them as functions.

- In the algebraic look, if you plug in a value for x you may get the same result, but they are still not equal as functions, because the value may not hold for all values of x.

In reference to high school teaching:
$\frac{x-1}{x^2-1}=\frac{x-1}{(x-1)(x+1)}=\frac{1}{x+1}$ This is often said in high school, but as teachers we should note that this is only the can when $x \neq 1$, because if we plug that into our original we get a 0 denominator, but in the final equation it works out fine.

Question: in what sense are equivalent ration functions equal as actual functions?

- The domain
- The intersection of the their domains

Two ratios of polynomials are equivalent if they are equal as functions on all shared points (where they are both defined) of the domain.