

**MATH 301/501 HOMEWORK-4 DUE AT THE BEGINNING OF CLASS ON
THURSDAY, OCT. 2**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions.

- (1) Let a, b denote elements in a Euclidean domain and assume a and b are both non-zero.

From the Euclidean algorithm we have:

$$\begin{aligned} a &= bq_1 + r_1, & \nu(r_1) &< \nu(b), \\ b &= r_1q_2 + r_2, & \nu(r_2) &< \nu(r_1), \\ r_1 &= r_2q_3 + r_3, & \nu(r_3) &< \nu(r_2), \end{aligned}$$

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$$\begin{aligned} r_{k-2} &= r_{k-1}q_k + r_k, & \nu(r_k) &< \nu(r_{k-1}), \\ r_{k-1} &= r_kq_{k-1}. \end{aligned}$$

- (a) Show that r_k is the GCD of a and b .
- (b) We've assumed that the process terminates. Can you say why?
- (2) Use the Euclidean Algorithm to find the GCD of:

$$2x^3 - 2x^2 + 3x$$

and

$$-2x^2 - 3x.$$

- (3) Use the Euclidean Algorithm to find the GCD of:

$$x^{10} - 3x^9 + 3x^8 - 11x^7 + 11x^6 - 11x^5 + 19x^4 - 13x^3 + 8x^2 - 9x + 3$$

and

$$x^6 - 3x^5 + 3x^4 - 9x^3 + 5x^2 - 5x + 2.$$