

**MATH 515 HOMEWORK–8 DUE AT THE BEGINNING OF CLASS ON
WEDNESDAY, DECEMBER 10 OR IN MATH 4-105 ON MONDAY, DECEMBER 15.**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions.

- (1) (a) State our definition of congruence.
(b) Using this definition, prove that two triangles with corresponding sides of equal length must be congruent.
- (2) In class we proved that any isometry of the plane with a fixed point must be a rotation or a reflection, (and, hence, a composition of reflections). Say as much as you can about isometries with no fixed points. Can you come up with any "new" isometries, distinct from those defined in class?
- (3) We want to prove that a circle is the plane figure that yields the maximum area for a fixed perimeter. In this problem, prove that the plane figure that yields the maximum area for a fixed perimeter must be *convex*.
- (4) Take a line segment with endpoints A and B . Join A and B by a curve γ as described in class. Suppose that, for any point C on γ , $\angle ABC$ measures 90° . Prove that γ is a semicircle, with diameter \overline{AB} .