MATH 513 HOMEWORK 1, SPRING 2019

DUE AT THE BEGINNING OF CLASS ON WEDNESDAY, FEBRUARY 6

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

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(1) Prove that, for every natural number n, the number $9^n - 4^n$ is a multiple of 5.

(2) Consider the inductive proof below, and explain the flaw in the argument. It might help to think about the statement of the Principle of Mathematical Induction.

Statement: For each natural number n, let $P(n)$ be the statement: For every collection of n cats, all cats in the collection are the same color.

Proof: Consider any set with one cat. Since there is only one cat in the set, certainly $P(1)$ is true.

Suppose, for some natural number k, that every set of k cats consists of cats of the same color.

Now take a set of $k+1$ cats. Remove one cat from the set. Since k cats remain, all of these cats are the same color. Put this cat back and remove a different cat. Since k cats remain, all of these cats are the same color. Hence, all of the original $k+1$ cats must be the same color, so the statement $P(k+1)$ is true.

By induction, $P(n)$ is true for all n, so all cats are the same color.

(3) Prove that, for every natural number n, we have:

$$(2)(6)(10)\cdots(4n-2) = \frac{(2n)!}{n!}.$$

Date: February 2, 2019.

(4) Prove that, for every natural number n,
$$(\cos x + i \sin x)^n = \cos(nx) + i \sin(nx).$$
You can use standard trigonometric identities without proof.

(5) Use induction to prove that, if $1+x > 1$ then, for each natural number n, $(1+x)^n \geq 1+nx$.