

**MATH 512 HOMEWORK-6 DUE AT THE BEGINNING OF CLASS ON THURSDAY,  
MARCH 12.**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. All your work should be clear and grammatically correct, in addition to mathematically sound. Solutions with little or no justification will receive little or no credit.

- (1) If you haven't already, read sections 7.1-7.3
- (2) Do exercise 7.8. You may already have most of this in your notes.
- (3) Let  $u$  denote a unit in  $\mathbb{Z}/m\mathbb{Z}$ . We proved in class that  $u$  is also an  $n$ th root of one in  $\mathbb{Z}/m\mathbb{Z}$ , for some  $n$ . In class it was conjectured that: given a unit  $u$  in  $\mathbb{Z}/m\mathbb{Z}$ ,  $u^{\phi(m)} = 1$ , where  $\phi(m)$  denotes the number of positive integers less than  $m$ , which are relatively prime to  $m$ . ***Prove this conjecture.*** An idea for the proof is given in exercise 7.9.
- (4) Read chapter 9.
- (5) Do exercises 9.2, 9.5, and 9.13. Exercise 9.2 asks you to prove that the units in the polynomial ring  $K[x]$  are the non-zero, constant polynomials.