

**MATH 301/501 HOMEWORK-4 DUE AT THE BEGINNING OF CLASS ON
TUESDAY, OCTOBER 13.**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

- (1) (a) Give the domain and range of the function $f(x) = \sin x$.
 (b) Is this function injective? Explain.
 (c) What is the definition or meaning of the function $g(x) = \sin^{-1} x$? Give some examples.
 (d) Give the domain and range of the function $g(x) = \sin^{-1} x$.
 (e) How are the $\sin^{-1} x$ function and its domain related to are study of equivalence relations and equivalence classes.
- (2) Let a, b denote elements in a Euclidean domain and assume a and b are both non-zero.

From the Euclidean algorithm we have:

$$\begin{aligned} a &= bq_1 + r_1, & \nu(r_1) &< \nu(b), \\ b &= r_1q_2 + r_2, & \nu(r_2) &< \nu(r_1), \\ r_1 &= r_2q_3 + r_3, & \nu(r_3) &< \nu(r_2), \end{aligned}$$

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$$\begin{aligned} r_{k-2} &= r_{k-1}q_k + r_k, & \nu(r_k) &< \nu(r_{k-1}), \\ r_{k-1} &= r_kq_{k-1}. \end{aligned}$$

- (a) Show that r_k is the GCD of a and b .
- (b) We've assumed that the process terminates. Can you say why?

(3) Use the Euclidean Algorithm to find the GCD of:

$$2x^3 - 2x^2 + 3x$$

and

$$-2x^2 - 3x.$$

(4) Use the Euclidean Algorithm to find the GCD of:

$$x^{10} - 3x^9 + 3x^8 - 11x^7 + 11x^6 - 11x^5 + 19x^4 - 13x^3 + 8x^2 - 9x + 3$$

and

$$x^6 - 3x^5 + 3x^4 - 9x^3 + 5x^2 - 5x + 2.$$