

**MATH 301/501 HOMEWORK-1 DUE AT THE BEGINNING OF CLASS ON
THURSDAY, SEPT. 10**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions. Solutions with little or no justification will receive little or no credit.

- (1)
 - (a) Write a clear, concise definition of an algebraic number.
 - (b) Give an example of a complex number that is algebraic, and prove that it is algebraic.
 - (c) Determine whether or not $\sqrt{1 + \sqrt{3}}$ is algebraic, and prove your result.

- (2)
 - (a) Write the definition of a group G with an operation denoted $*$. (Don't assume that this operation is addition or multiplication.)
 - (b) *Prove* that the identity element of the group must be unique.

- (3) For each statement, either prove the statement or provide a counter-example.
 - (a) The rational numbers are closed under addition.
 - (b) The irrational numbers are closed under addition.
 - (c) The rational numbers are closed under multiplication.
 - (d) The irrational numbers are closed under multiplication.

- (4) At the end of class we discussed the set of $n \times n$ matrices. Take this set, and assume that the entries of the matrices are real numbers.
 - (a) Show that this set satisfies the properties of a *ring with an identity*.
 - (b) Show that the multiplication in this ring is *not* commutative.
 - (c) Give an element in the ring that has no multiplicative inverse. Explain.
 - (d) Explain why the set of all 2×3 matrices and 3×2 matrices does not form a ring.