

**MATH 301/501 HOMEWORK-7 DUE AT THE BEGINNING OF CLASS ON  
THURSDAY, DECEMBER 11.**

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions.

- (1) In class we looked at the *dihedral group*  $D_n$ , which may be described geometrically as the *group* of symmetries of a regular  $n$ -gon.
  - (a) In class some students found 2 generators of  $D_3$ . Find 2 generators, and show, algebraically, that they generate the group.
  - (b) Do the same thing for  $D_4$ .
  - (c) Prove that the group  $D_n$  has 2 generators.
- (2)
  - (a) Clearly state the definition of a distance function on a set.
  - (b) Prove that the 2-dimensional Euclidean distance satisfies the definition of a distance function on  $\mathbb{R}^2$ .
- (3) In class we proved that a reflection is an isometry. Prove that a rotation about the origin is an isometry.
- (4)
  - (a) Give an example of a function studied in high school mathematics whose graph has translational symmetry. If the graph has other symmetries, discuss these as well.
  - (b) Give an example of a different graph, studied in high school math, which has non-trivial, (not just 360 degrees), rotational symmetry. Is this the graph of a function?
  - (c) What can you say about rotational symmetries of graphs of real-valued functions on  $\mathbb{R}$ ? Is it possible to find rotational symmetries through angles of arbitrary degree? If so, explain. If not, explain the limitations.
  - (d) Give an example of a different graph, studied in high school mathematics, which has reflection symmetry. Is this the graph of a function?
- (5) More may be posted by the end of the day on Thursday.