MATH 301/501 HOMEWORK–1 DUE AT THE BEGINNING OF CLASS ON TUESDAY, SEPT. 16

One goal for this course is for you to develop your skill in effectively communicating mathematics. With this in mind, you should clearly write up your solutions.

(1) (a) Write a clear, concise definition of a function.
(b) Write a clear, concise definition of an injective function.
(c) Write a clear, concise definition of a surjective function.
(d) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ that is injective but not surjective.
(e) Give an example of a function $f : \mathbb{Z} \to \mathbb{Z}$ that is surjective but not injective.
(f) Give an example of a bijective function $f : \mathbb{Z} \to \mathbb{Z}$.
(g) Give an example of a bijective function $f : \mathbb{N} \to \mathbb{Z}$.
(h) Give an example of a bijective function $f : (0, 1) \to \mathbb{R}$.
(i) Make sure you justified each of your examples.

(2) Recall from class that a function f from a group $(G_1, *)$ to a group $(G_2, \cdot)$ is called a group *homomorphism* if, for every $a, b \in G_1$, $f(a * b) = f(a) \cdot f(b)$.

(a) Find conditions on m and b so that the function $g : (\mathbb{R}, +) \to (\mathbb{R}, +)$ defined by $g(x) = mx + b$ is a group homomorphism.

(b) Is the function $h(x) = x^2$ a group homomorphism from the additive group of real numbers to the additive group of real numbers?

(c) Is the function $f(x) = \log x$ a group homomorphism? Explain!

(d) To be posted.
(e) To be posted.

(3) In the notes for the first class Gina makes reference to the notion of an *ordered field*. An ordered field is a set F closed under two operations, $+$ and $\cdot$, satisfying the field axioms stated in class, with a *relation*, denoted $<$, that satisfies the following properties:

- For every x and y in F, exactly one of the following is true: $x < y$, $y < x$, or $x = y$.
- The relation is *transitive*: $x < y$ and $y < z$ imply that $x < z$.
- $x < y$ implies that $x + z < y + z$.
- $x < y$ and $0 < z$ implies $xz < yz$.

(a) Let $-x$ denote the additive inverse of x. Use the defining properties of an ordered field to prove that: if $x < y$ then $-y < -x$.
(b) Prove that $0 < 1$, where 0 denotes the additive identity and 1 denotes the multiplicative identity.
(c) Prove that if $0 < x < y$ then $0 < \frac{1}{y} < \frac{1}{x}$.
(d) Prove that the complex numbers do not form an ordered field.