MAT 132 FINAL EXAM 1 INFORMATION

The final exam for Math 132 will be held on Friday, May 10, 2:00-4:30. The location of the exam depends on your section:

Sections 1-7 will be in JAVITS 100.

Sections 8-10 will be in JAVITS 110.

Section 11 will be in JAVITS 103.

You may NOT use a calculator. You may NOT use any books or notes. Please show up at least 5 minutes early to ensure that everyone has the full $2\frac{1}{2}$ hours to work on the exam.

The exam is COMPREHENSIVE. It covers ALL topics from the course, including topics from the first and second midterms. To determine what old material to review, look at the old web pages which describe what to expext on the first and second midterms. You should review all of the material described on these pages.

The following is a list of NEW TOPICS (since the second midterm) which you should review for the final exam.

- Understand the difference between a sequence and a series. What does it mean for a sequence to converge/diverge? What does it mean for a series to converge/diverge?
- A series is "geometric" if each term is obtained from the previous term by multiplying by a fixed number, r. A geometric series has the form

  $$\sum_{n=0}^{\infty} ar^n = a + ar + ar^2 + ar^3 + \cdots$$

  If $|r| < 1$, it converges to $\frac{a}{1-r}$. If $|r| \geq 1$, it diverges.
- The "harmonic series" is the series

  $$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots$$

  The harmonic series DIVERGES.

- A p-series is the result of raising all the terms of the harmonic series to a fixed power:
$$\sum_{n=1}^{\infty} \frac{1}{n^p} = 1 + \frac{1}{2^p} + \frac{1}{3^p} + \frac{1}{4^p} + \cdots$$
When $p = 1$, this is just the harmonic series. The p-series converges if $p > 1$ and diverges if $p \leq 1$. We proved this fact using the INTEGRAL TEST by comparing to the improper integral $\int_1^\infty \frac{1}{x^p}$ dx. The cases $p = 1$ and $p = 2$ were done in a discovery-learning project.
- You will be given several series and asked to determine whether they converge. So you should understand and practice using the following convergence tests: The COMPARISON TEST, the INTEGRAL TEST, the RATIO TEST, the ALTERNATING SERIES TEST. Also remember that if the terms of a series do not go to zero, then the series could not possibly converge.
- A "power series" is an infinite-degree polynomial,
$$f(x) = \sum_{n=0}^{\infty} C_n (x-a)^n = C_0 + C_1(x-a) + C_2(x-a)^2 + \cdots$$
The domain of $f(x)$ is called the INTERVAL OF CONVERGENCE. It is the set of all numbers which when plugged in for x result in a convergent series. This domain is always an interval centered at a (possibly with radius 0 or ∞, and possibly including one or both endpoints).
- Know how to use the RATIO TEST to determine the radius of convergence of a power series. Once you know the radius of convergence, you must check convergence at the endpoints to determine the interval of convergence.
- Sometimes a power series equals a familiar function on it's interval of convergence, and sometimes not.
- The Taylor Series for $f(x)$ at a is the following power series:
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots$$
Practice using this formula to find the Taylor series of a given function.
- In the special case where $a = 0$, the Taylor series is called the Maclaurin series:
$$\sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \cdots$$

- The only power series which could possibly equal $f(x)$ on an interval about a is the Taylor series. However, once you compute the Taylor series, you still need to check that it equals the function on its interval of convergence. This can often be done by using TAYLOR'S INEQUALITY (page 614). However, this skill will NOT be tested on the final.
- A Taylor polynomial is the result of ending a Taylor series after only finitely many terms:

$$f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n.$$

  The Taylor polynomial is contrived to have the same first n derivatives at a as the function $f(x)$ does.
- You should memorize the Taylor series for $\frac{1}{1-x}$, e^x, $\sin x$ and $\cos x$. These are all found on page 618.
- You should memorize the Taylor series for $(1+x)^k$. This is found on page 623.
- Instead of computing a Taylor series from scratch, it is often possible to start with a Taylor series you know, and do something to it (integrate it, differentiate it, or make a substitution). Practice this skill
- Practice finding power series solutions to differential equations (section 8.10).