Solutions of Sections 3.4 and 3.5

Section 3.4

4. $g(t) = 4\sec t + \tan t$

Solution:

$$\frac{d}{dt}g(t) = 4\sec t\tan t + \sec^2 t$$

6. $y = e^u(\cos u + cu)$

Solution:

$$\frac{d}{du}y = e^u(\cos u + cu) + e^u(-\sin u + c)$$

8. $y = \dfrac{\sin x}{1+\cos x}$

Solution:

$$\begin{aligned}\frac{d}{dx}y &= \frac{(\sin x)'(1+\cos x) - \sin x(1+\cos x)'}{(1+\cos x)^2}\\ &= \frac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}\\ &= \frac{\cos x + \cos^2 x + \sin^2 x}{(1+\cos x)^2}\\ &= \frac{1}{1+\cos x}\end{aligned}$$

10. $y = \dfrac{\tan x - 1}{\sec x}$

Solution:

$$\begin{aligned}\frac{d}{dx}y &= \frac{(\tan x - 1)'\sec x - (\tan x - 1)(\sec x)'}{\sec^2 x}\\ &= \frac{\sec^2 x\sec x - (\tan x - 1)\sec x\tan x}{\sec^2 x}\\ &= \frac{\sec x + \sec x\tan x}{\sec^2 x}\\ &= \frac{1+\tan x}{\sec x}\end{aligned}$$

12. $y = \csc\theta(\theta + \cot\theta)$

Solution:

$$\begin{aligned}
\frac{d}{d\theta}y &= (\csc\theta)'(\theta+\cot\theta)+\csc\theta(\theta+\cot\theta)' \\
&= (-\csc\theta\cot\theta)(\theta+\cot\theta)+\csc\theta(1-\csc^2\theta) \\
&= -\theta\csc\theta\cot\theta-\csc\theta\cot^2\theta-\csc\theta-\csc^3\theta \\
&= -\theta\csc\theta\cot\theta-2\csc^3\theta
\end{aligned}$$

26. Find the points on the curve $y = (\cos x)/(2+\sin x)$ at which the tangent is horizontal.

Solution: The points at which the tangent is horizontal are those points x satisfying $y'(x) = 0$. We have

$$y'(x) = \frac{-\sin x(2+\sin x)-\cos x\cos x}{(2+\sin x)^2} = \frac{-2\sin x-1}{(2+\sin x)^2},$$

so $y'(0) = 0 \iff -2\sin x - 1 = 0 \iff \sin x = 1/2 \iff x = 2k\pi + \pi/2 \pm \pi/3, \quad k = 0, \pm1, \pm2, \cdots.$

28. Let $f(x) = x - \sin x, 0 \le x \le 2\pi$. On what interval is f concave upward?

Solution: The interval on which f is concave upward is the interval on which $f''(x) \ge 0$. We have $f''(x) = \sin x$, so solve $f''(x) = \sin x \ge 0$ we have $0 \le x \le \pi$.

37. $\lim\limits_{x\to0}\dfrac{\tan 4x}{x}$

Solution:

$$\lim_{x\to0}\frac{\tan 4x}{x} = \lim_{x\to0}\frac{\sin 4x}{x\cos 4x} = \lim_{x\to0}\frac{\sin 4x}{4x}\lim_{x\to0}\frac{4}{\cos 4x} = 1\times4 = 4$$

38. **Solution:**

$$\begin{aligned}
\lim_{x\to0} x\cot x &= \lim_{x\to0} x\frac{\cos x}{\sin x} \\
&= \lim_{x\to0}\frac{x}{\sin x}\cos x \\
&= \lim_{x\to0}\frac{x}{\sin x}\cdot\lim_{x\to0}\cos x \\
&= 1
\end{aligned}$$

Section 3.5

8. $F(x) = (x^2 - x + 1)^3$
Solution: $F'(x) = 3(x^2 - x + 1)(x^2 - x + 1)' = 3(x^2 - x + 1)^2(2x - 1)$.

10. $f(t) = \sqrt[3]{1 + \tan t}$
Solution:

$$\begin{aligned} f'(t) &= \left((1 + \tan t)^{1/3}\right)' \\ &= \frac{1}{3}(1 + \tan t)^{-2/3} \cdot (1 + \tan t)' \\ &= \frac{1}{3}(1 + \tan t)^{-2/3} \sec^2 t \end{aligned}$$

12. $y = a^3 + \cos^3 x$
Solution: $y' = (a^3 + \cos^3 x)' = 3\cos^2 x \cdot (\cos x)' = -3cox^2 x \sin x$

14. $y = 4 \sec 5x$
Solution: $y' = 4(\sec 5x)' = 4 \sec 5x \tan 5x \cdot (5x)' = 20 \sec 5x \tan 5x$

16. $y = e^{-5x} \cos 3x$
Solution: $y' = (e^{-5x})' \cos 3x + e^{-5x}(\cos 3x)' = -5e^{-5x} \cos 3x - 3e^{-5x} \sin 3x$

20. $y = 10^{1-x^2}$
Solution: $y' = 10^{1-x^2} \ln 10 \cdot (1 - x^2)' = -20 \cdot \ln 10 \cdot 10^{1-x^2}$

24. $y = \dfrac{e^{2u}}{e^u + e^{-u}}$
Solution:

$$\begin{aligned} y' &= \frac{(e^{2u})'(e^u + e^{-u}) - e^{2u}(e^u + e^{-u})'}{(e^u + e^{-u})^2} \\ &= \frac{2e^{3u} - 2e^u - e^{3u} + e^u}{(e^u + e^{-u})^2} \\ &= \frac{e^{3u} - e^u}{(e^u + e^{-u})^2} \end{aligned}$$

26. $y = \tan^2(3\theta)$
Solution: $y' = 2\tan(3\theta) \cdot (\tan 3\theta)' = 6 \tan 3\theta \sec^2 3\theta$

28. $y = \sin(\sin(\sin x))$
Solution: $y' = \cos(\sin(\sin x))\cos(\sin x)\cos x$

30. $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$
Solution:

$$\begin{aligned} y' &= \left(\left(x + (x + x^{1/2})^{1/2}\right)^{1/2}\right)' \\ &= \frac{1}{2}\left(x + (x + x^{1/2})^{1/2}\right)^{-1/2} \cdot \left(x + (x + x^{1/2})^{1/2}\right)' \\ &= \frac{1}{2}\left(x + (x + x^{1/2})\right)^{-1/2} \cdot \left(1 + \frac{1}{2}(x + x^{1/2})^{-1/2} \cdot (1 + \frac{1}{2}x^{-1/2})\right) \end{aligned}$$