1 List of the many objects (symbols) in the series of papers on Tensor structures by Kazhdan and Lusztig

Here we list , in rough alpha-betical order and also order of appearance, the objects (or symbols) used by Kazhdan and Lusztig in their series of paper "Tensor Structures Arising from Affine Lie Algebras, I-IV", published in the Journal of the American Mathematical Society, 1993-1994.

a-isomorphism (424) a morphism between objects in $\mathscr{O}_{\kappa_0}$ whose ker and coker are both equivalent to zero

$\mathscr{A}_k$ (section 27.1, p.390), the full subcategory in $\mathcal{O}$ of modules having a Weyl filtration or other projectivity or ext conditions

A (1.1) a commutative algebra with 1

admissible filtration (pp. 391, 404) is a property satisfied by objects in $\mathcal{A}_\kappa$

$\dot{A}$, $\dot{A}'$ (17.2, p.993) are spaces of regular functions on $\dot{C}$ and $\dot{C}'$ respectively

A_∞, A'_∞ (15.4) are space of regular functions over D and D' respectively

$A, \tilde{A}, A', \tilde{A}'$ (9.3, p.951;10.3,p.957) are, respectively, spaces of regular functions $\mathbb{C}[\underline{\mathscr{V}}], \mathbb{C}[\mathscr{V}], \mathbb{C}[\underline{\mathscr{V}}'], \mathbb{C}[\mathscr{V}']$

A_0 (10.2, p.957) $= \mathbb{C}[H] = \mathbb{C}[PGL_2(\mathbb{C})] = \mathbb{C}[GL_2(\mathbb{C})]_{\mathbb{C}^*}$, the subalgebra of $\mathbb{C}[GL_2(\mathbb{C})]$ consisting of functions invariant under the scaling action $(g_{ij} \mapsto \lambda g_{ij})$ of $\mathbb{C}^*$.

(a_{ij}) (19.1) a Cartan matrix

α_i (19.1) simple roots

A (19.1, p.336) $= \mathbb{C}[[\varpi]]$

$\mathfrak{A}$ (19.1) the universal envelope of $\mathfrak{g}$

A_{V_1,V_2,V_3} (19.10, p.340, 339) the associativity isomorphism

$$R(t_{12}, t_{23}) : V_1 \otimes V_2 \otimes V_3 \to V_1 \otimes V_2 \otimes V_3$$

$\mathscr{A}_\kappa$ (§27, p.388) is the subcategory of $\mathscr{O}_\kappa$ consisting of modules with Weyl filtrations

$\mathscr{A}_\kappa^t$ (391) is defined as $\mathscr{A}_\kappa \cap \mathscr{O}_\kappa^t$

$\check{s}_a^\kappa$ (393) is defined as the nonzero morphism $\mathbf{V}_a^\kappa \to D(\mathbf{V}_{\bar{a}}^\kappa)$ defined by s_a^κ (Lem.2.32 (c)).

$\mathscr{A}_\varphi$ (401) a subcategory of $\mathscr{O}_\varphi$.

$\alpha(\kappa) = \varphi(\kappa) \circ \beta(\kappa)$

α (406, §30) is defined as $q_V \circ p_V$ where $V \in \mathscr{O}_\kappa$, $p_V : \mathscr{F}(V) \to V$ and $q_V : V \to \mathscr{G}(V)$

a (407, §31) associativity isomorphism for $(V_1 \dot{\otimes} V_2)\dot{\otimes} V_3)$, defined as in §18.2, for objects $V's$ in $\mathscr{A}$.

$\hat{a}$ (407, §30) associativity isomorphism for fusion tensor products of objects in $\mathscr{O}_\infty$.

A_{an} (407, §30) ring of R_∞-valued analytic germs at $0 \in \mathbb{C}$

a_0, a_1 (409) linear isomorphisms from $\langle \mathscr{N} \rangle \to \mathscr{N}$ given by the asymtotics

$$a_0(n(t)) = \lim_{t \to 0} (t^{\Omega_{12}/\kappa} n(t));$$

$$a_1(n(t)) = \lim_{t \to 1} ((1-t)^{\Omega_{23}/\kappa} n(t)).$$

It turns out that (corollary, p.409) $a^\infty = a_1 \circ a_0^{-1}$.

A (433) is defined as $\mathbb{C}[v, v^{-1}]$

B (8.1, p.942) a commutative $\mathbb{C}$-algebra with 1, as and alternative base ring to $\mathbb{C}$

B, B' (9.7, p.952) B is a $\mathbb{C}$-algebra with a $\mathbb{C}$-homomorphism $A \to B$; $B' := B \otimes_A A'$.

B_1 (9.13, p.954) another ring, for studying the base change of Y_{Δ_B}

B_∞, B'_∞ (15.13, p.980) $B_\infty = \mathbb{C}[t]$ and $B'_\infty = \mathbb{C}[t,p,q]/(pq-t)$

B_n, B'_n (15.13) $B_n = B_\infty/(t^n)$, and $B'_n = B'_\infty/(t^n)$

(b_{ij}) (19.1) inverse of Cartan matrix (a_{ij})

B_{V_1,V_2,V_3,V_4} **(or B')** (19.10, p.340) are maps from $(V_1 \otimes V_2) \otimes (V_3 \otimes V_4)$ to $V_1 \otimes (V_2 \otimes V_3) \otimes V_4$; with the target tensor product associated in the two possible ways.

$\nabla(b), \Delta(b), \mathscr{F}(\Delta)$ (pp.390-391) are, respectively, A-modules and category defined in C.M.Ringel, *The category of modules with good filtrations ove a quasi-hereditary algebra has almostsplit sequences* , Math. Z. 208 (1991), pp. 209-223

$\beta(\kappa)$ (404) is a morphism: $M(\kappa) \to M_1(\kappa)$ such that

B (405)

$\beta(V_1, V_2)$ (407) is an isomorphism $\mathfrak{g}(V_1) \otimes \mathfrak{g}(V_2) \to \mathfrak{g}(V_2 \dot{\otimes} V_2)$ where V_1, V_2 are Weyl modules; It defines an isomorphism between functors θ_1, θ_2.

$\tilde{\beta}$ (407) is an isomorphism: $\tilde{W}_1 \dot{\otimes} \tilde{W}_2 \tilde{\to} (W_1 \tilde{\otimes} W_2)^\kappa$, where $W_1, W_2 \in \mathscr{D}$

$(b), (b^*)$ (435) dual bases of $\mathbf{f}_\nu$

$\dot{C}$**,** $\dot{C}'$ (17.1, p.992) are, respectively, the Riemann sphere with three points 0, 1, and ∞ removed, and the variety of (punctured) quadrics $\{(t,p,q) \in \mathbb{C}^3 | pq = t; p, q \neq 1\}$ over it

$\chi^t(V, a)$ (p.396) is the alternating sum $\sum_{i \in \mathbb{N}} (-1)^i \cdot dimExt^i_{\mathcal{O}^t_\kappa}(V, D(\mathbf{V}^\kappa_a))$

$\mathscr{C}$**,** $\mathscr{C}^+$**,** $\mathscr{C}^0$ (3.7, p.932) are abelian categories, with objects complex-vector spaces V with a given decomposition $V = \bigoplus_{\lambda \in \mathbb{C}} ({}_\lambda V)$, and $\tilde{\mathfrak{g}}$-module structure (respectively, $\tilde{\mathfrak{g}}^+$-module and $(\mathfrak{g} \oplus \mathbb{C}\mathbf{1})$-module structure for $\mathscr{C}^+$ and $\mathscr{C}^0$) such that

1. $\epsilon^n c({}_\lambda V) \subset {}_{\lambda - n}V$ for all $c \in \mathfrak{g}$ and $n \in \mathbb{Z}$ (respectively, $n \in \mathbb{N}$ and $n = 0$)
2. $\mathbf{1}$ acts as a scalar $\kappa - h$
3. the action of $\mathfrak{g}$ on each piece ${}_\lambda V$ is locally finite (i.e. has finite dimensional orbit on each vector in V)

And irreducible object in $\mathscr{C}^-$ is isomorphic to some $\mathscr{V}_a^\kappa(\lambda)$ where $\lambda \in \mathbb{C}$.

$\mathscr{C}$ is the ring of functions $\frac{B_n[p,q]}{(pq-t)}[(p-1)^{-1},(q-1)^{-1}]$

C_{V_1,V_2} (19.12) the commutativity isomorphism: $C_{V_1,V_2}(x \otimes y) = e^{i\pi\varpi t}(y \otimes x)$, where the exponential is defined as a Taylor series.

$c_i(F)$ (26.4, p.385) is a linear operator: $\bar{Y} \to Y/\Delta Y : c_i(F)c(x' \otimes x_i \otimes x'') = \pi(x' \otimes (Fc)x_i \otimes x'')$ where $1 \le i \le n$ and $F \in A((\epsilon))$.

$C_{a,b,c}$ (393) is defined as $dimHom(\mathscr{V}_a \otimes \mathscr{V}_b \otimes \mathscr{V}_c, \mathbb{C})$

$C_{\bar{a},b,\bar{c}}$ (429) is defined as $dim_{\mathbb{C}}(\mathscr{V}_c^\lambda)$

$\mathscr{C}_\kappa$ (436) is a braided category (of modules over a quantum group)

c_i (438)

$\mathscr{D}_0$ (p.958) it is the complex vector subspace of $Der(\tilde{A})$ generated by $\theta_{0,s}$, $\theta_{+1,s}$, $\theta_{-1,s}$ for $s \in S$ (S=the configuration of points on the Riemann sphere)

$\mathscr{D}_1$ (p.958) it is a subspace of the set of derivations, which commute with the action of H, of the algebra $\tilde{A} = \mathbb{C}[\mathscr{V}]$ on the "lifted" configuration space $\mathscr{V}$ of marked points on the Riemann sphere, so that it maps surjectively $\mathcal{D}_1 \to Der(A)$ onto the space of derivations on the moduli space $\underline{\mathscr{V}}$

$\mathscr{D}_2$ (10.12, p.961) it is the Lie subalgebra of $\mathscr{D}_1$ consisting of derivations sending the subalgebra $A \subset \tilde{A}$ to zero, i.e. they are derivations along the fibres of the projection $\mathscr{V} \to \underline{\mathscr{V}}$; It is also an A-Lie algebra: the Lie algebra structure and the A-module structure on $\mathscr{D}_2$ are compatible. We have (12.11, p.969) $Der(A) = \mathscr{D}_1/\mathscr{D}_2 \to End_{\mathbb{C}}(Y/\Delta Y)$.

D, D' (15.2, p.977) are, respectively, the Riemann sphere with two points 1 and ∞ removed, and the variety of (punctured) quadrics $\{(t,p,q)|pq = t; p,q \neq 1\}$ over it

$\dot{\Delta}_4$ (17.5,p.994) the Lie algebra $\dot{A}' \otimes \mathfrak{g}$

$D(V)$ (1.16) the duality functor of $\mathcal{O}_\kappa$ defined by $D(V) = {}^dV^\#(\infty)$, where ${}^dV = Hom_A(V, A)$; That is to say, $D(V)$ is the subspace of linear forms $V \to \mathbb{C}$ having zero values on $Q_N^\# V$ for some N¿1, with the inherited action of $\hat{\mathfrak{g}}$

${}^dV$ (1.16) $= Hom_A(V, A)$

Δ (9.9, p.952) is the A-Lie algebra $A' \otimes \mathfrak{g}$ with bracket $[fc, f'c'] = ff'[c, c']$. There is a natural homomorphism of A-Lie algebras $\Delta \to \hat{\mathfrak{g}}_A^S$ given by $fc \mapsto \sum_{s \in S} \delta_s({}^sfc)$

δ_s (1.6, p.908) is the Lie algebra embedding $\hat{\mathfrak{g}}_A \to \hat{\mathfrak{g}}_A^S$ extending the simple embedding $\mathfrak{g} \to \mathfrak{g}^S$

Δ_B (9.9, p.953) is the B-Lie algebra $\Delta_B = B \otimes_A \Delta (= B' \otimes \mathfrak{g})$. There is a homomorphism of B-Lie algebras $\Delta_B \to B \otimes_A \hat{\mathfrak{g}}_A^S \to \hat{\mathfrak{g}}_B^S$. There is an exact sequence (9.10(a),p.953)

$$0 \to B \otimes \mathfrak{g} \xrightarrow{\alpha} \Delta_B \oplus (\hat{\mathfrak{g}}_B^S)^+ \xrightarrow{\alpha'} \hat{\mathfrak{g}}_B^S \to 0$$

where $(\hat{\mathfrak{g}}_B^S)^+ = B[[\epsilon]]^S \otimes \mathfrak{g}$ (c.f. Lemma 9.8, 9.10)

$Der(A_0)$ (10.2, p.957) The Lie algebra of derivations of A_0; It has an A_0-basis $\{\theta_0, \theta_{-1}, \theta_1\}$ given by the formulae $\theta_{-1} = \theta^{12}, \theta_0 = \theta^{11} = -\theta^{22}, \theta_1 = -\theta^{21}$ where $\frac{\partial}{\partial g_{ij}} = \frac{g_{3-i,3-i}\theta^{ij} - g_{3-i,i}\theta^{3-i,j}}{g_{11}g_{22} - g_{12}g_{21}}$

$\mathscr{D}_0$ (p.958) it is the complex vector subspace of $Der(\tilde{A})$ generated by $\theta_{0,s}$, $\theta_{+1,s}$, $\theta_{-1,s}$ for $s \in S$ (S=the configuration of points on the Riemann sphere)

$\mathscr{D}_1$ (p.958) it is a subspace of the set of derivations, which commute with the action of H, of the algebra $\tilde{A} = \mathbb{C}[\mathscr{V}]$ on the "unquotiented" configuration space $\mathscr{V}$ of marked points on the Riemann sphere, so that it maps surjectively $\mathcal{D}_1 \to Der(A)$ onto the space of derivations on the moduli space $\underline{\mathscr{V}}$. $\mathscr{D}_1$ is mapped to $End_B(Y)$ by the map Λ', an extension of the map Λ taking the vector fields $\theta_{k;s}$ to Sugawara operators $1 \otimes \cdots L_{k;t} \cdots \otimes 1$

$\mathscr{D}_2$ (10.12, p.961) it is the Lie subalgebra of $\mathscr{D}_1$ consisting of derivations sending the subalgebra $A \subset \tilde{A}$ to zero, i.e. they are derivations along the fibres of the projection $\mathscr{V} \to \underline{\mathscr{V}}$; It is also an A-Lie algebra: the Lie algebra structure and the A-module structure on $\mathscr{D}_2$ are compatible. We have (12.11, p.969) $Der(A) = \mathscr{D}_1/\mathscr{D}_2 \to End_{\mathbb{C}}(Y/\Delta Y)$. It is mapped into $End_B(Y)$ by the Lie algebra map (lemma 11.3, p.963) Λ'' which is a restriction of Λ'.

δ_i (19.1) symmetrizer, i.e. $(\delta_i a_{ij})$ is a symmetric matrix, it is assumed to be positive-definite

$\mathscr{D}$ (§19, see 19.3,p.337) Drinfeld's category, with as objects free $\mathfrak{A}$-modules of finite rank over $\mathbf{A}$; and as morphisms $\mathfrak{A}$-linear maps. Its indecomposable objects are V_a where a is in the weight lattice (19.3). There is a structure of tensor category on $\mathscr{D}$ with commutativity (19.12) and associativity (19.10) isomorphisms constructed from (formal) Knizhnik-Zamolodchikov equations (19.8(a)) - see $R(\Pi_0, \Pi_1), or A_{V_1,V_2,V_3}$(19.10).

$\mathscr{D}$ (26.4) the category of finite dimensional complex vector spaces

$d_m(a)$ (393) is defined as $dim\mathbf{V}_{a,m}$

$D_{\alpha,m}(a)$ (393) is defined as $\sum_{c\in Y_\alpha(a)} d_m(c)$ when α is a dominant weight; it can be extended (p.393) to the whole weight lattice with the action of the Weyl group

$d_M^K(A,\Lambda)$ (399) is defined as $d_{M(K)}(A_K,\Lambda)$.

δ_a (411) is defined as $\varphi_a = \delta_a Id$, where $\delta_a \in R_\infty$. Its order of pole at κ plays a key role in the numerical criterion of rigidity in Prop. 31.3, p.412.

Δ (416) discriminant function on T

$\Delta_{4,n}$ (979) is defined as $A'_n \otimes \mathfrak{g}$

d_a (412) is defined as δ_a^{-1}. It turns out (31.6, p.413) that $d_a(\kappa) = \frac{v^n - v^{-n}}{v - v^{-1}}$

$\mathscr{D}_R$ (421) category of finitely generated R-modules

$d(a,\kappa)$ (430) is defined as $dimL_a$

e (405) is the natural projection $e : M = M_N \to M_N/M_{N-1} = \mathbf{1}$; We have $e \circ \tilde{i} = (x-\kappa)^{N-1}$.

$e_{\mathbf{V}_a}$ (412) is a morphism $\mathbf{V}_a^{\hat{\kappa}} \dot{\otimes} D(\mathbf{V}_a^{\hat{\kappa}}) \longrightarrow \mathbf{1}$.

η (423) a morphism: $\mathbf{V}_{a+b-\alpha_i} \to W$

E_i^V (426)

$\bar{E}_i^W$ (428) defined analogously to E_i^V

$\bar{\mathscr{E}}(\kappa)$ (430) is a category of graded finite-dimensional vector spaces

$E_i^{(p)}$ (436) as an operator $(M^*)^\lambda \to (M^*)^{\lambda+pi'}$, is defined as:

$$(E_i^{(p)} m^*)(m) = (-1)^p v^{p\lambda(i)+p(p-1)} m^*(E_i^{(p)} m)$$

for $m^* : M^{-\lambda} \to R, m \in M^{-\lambda+pi'}$

f_s **and** f_s^k (sec. 9.5) $f_s := \frac{1}{\gamma_s^{-1}(z)} \in \mathbb{C}$ is the reciprocal of the value of the local chart of a point z on the projective line $P(1) \cong \mathbb{C}$. The set $\{1, f_s^k(s \in S; k \geq 1)\}$ form a basis of A' over A (Lemma 9.6, p.951)

${}^sf$ (9.4) is a homomorphism of A-algebras: $A' \to A((\epsilon))$, which is an expansion along the extra parameter z of $\mathscr{V}'$ over $\mathcal{V}$ of a regular function $f \in A' = \mathbb{C}[\underline{\mathcal{V}'}]$ on the moduli space $\underline{\mathscr{V}'} = H \backslash \mathscr{V}'$ of the Riemann sphere with marked points

F^t (3.3) a finite set $\{a \in \mathbb{N}^I | \langle a, a+2 \rangle \leq t\}$ where t is an integer greater than (or equal to?) 1

f_s **and** f_s^k (9.5, p.951) $f_s := \frac{1}{\gamma_s^{-1}(z)} \in \mathbb{C}$ is the reciprocal of the value of the local chart of a point z on the projective line $P(1) \cong \mathbb{C}$. The set $\{1, f_s^k(s \in S; k \geq 1)\}$ form a basis of A' over A (Lemma 9.6, p.951)

ϕ (10.3) is a map: $H \times P^1 \to P^1$ given by the evaluation $\phi(\gamma, z) = \gamma(z)$. The vector fields θ_k act on the first factor γ, and we have $\theta_k(\phi) = z^{k+1} \frac{\partial \phi}{\partial z}$

$\tilde{f}(z)$ (19.8) is the rescaled function $z^{\varpi \Pi_0} f(z)$ where f(z) is a solution of the formal KZ equation (19.8(a)).

$\tilde{f}_n$ (19.8, p.339) Fourier coefficients of the series expansion of $\tilde{f} = \sum_{n \geq 0} \varpi^n \tilde{f}_n$. It is a function: $(0,1) \to {}^0V$ which extends to the interval $[0,1)$ (p.339).

$R_0(f), R_1(f)$ (p.339) are two maps from the space $\mathscr{X}$ of solutions of KZ equations (19.8(a)) to the representation space V. $R_0(f)$ is the rescaled asymtotic of f at 0, i.e.

$$R_0(f) = \lim_{z\to 0} \tilde{f} = \sum_{n\geq 0} \varpi^n \lim_{z\to 0} \tilde{f}_n(z) \in V.$$

$R_1(f)$ is a similar asymptotic at 1, where f is rescaled by $(1-z)^{-\varpi\Pi_1}$.

f_N (26.3) is defined as the natural map $\hat{W}(-\infty) \to W/G_N W$.

$f_i j$ (26.4) a function on $\mathscr{V}$

F_κ (404) field of fractions of R_κ

$\mathscr{F}$ (406, §30) is defined as the functor: $V \mapsto V(1)$ from $\mathscr{O}_\kappa$ to $\mathscr{D}$, the category of finite dimensional representations of the Lie algebra $\mathfrak{g}_F := \mathfrak{g} \otimes_{\mathbb{C}} F$ where κ is a ring morphism: $R \to F$ from ring R to a field F.

$F_{a,b}(\kappa)$ (424) is a rational function of Gamma factors

F_i^V (427)

$\bar{F}_i^W$ (428) defined analogously to F_i^V

$\mathbf{f}$ (433) is an algebra over $\mathbb{C}(v)$ with generators $\kappa_i : i \in I$ and relations

$$\sum_{p,p'\in\mathbb{N};p+p'=1-a_{ij}} (-1)^{p'} \frac{\kappa_i^p}{[p]!} \kappa_j \frac{\kappa_i^{p'}}{[p']!} = 0$$

${}_A\mathbf{f}$ (434) is the A-subalgebra of $\mathbf{f}$ generated by the divided powers $\kappa_i^{(p)} = \frac{\kappa_i^p}{[p]!}$

$\mathbf{f}_\nu$ (434) ???

${}_R\mathbf{f}$ (434) is an R-algebra defined by a change of scalar; where R is a commutative algebra over A

${}_R\mathscr{C}$ (434) is an abelian category of finitely generated R-modules with action of ${}_R\mathbf{f}$.

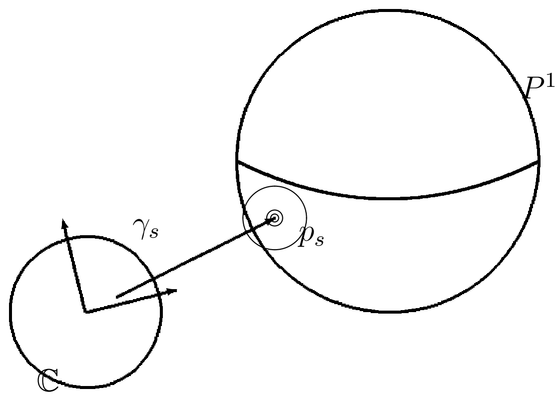
$F_i^{(p)}$ (436) is defined analogously to $E_i^{(p)}$

$\hat{\mathfrak{g}}^S, \tilde{\mathfrak{g}}^S$ are central extensions of $\mathfrak{g}^S = \overbrace{\mathfrak{g} \oplus \cdots \oplus \mathfrak{g}}^{S}$, i.e. they have one dimensional centres, and are not direct sums of affine Lie subalgebras

$g_{a,b}$ (20.6(a)): elements in $\mathbf{A}$, measuring the discrapency of the S'_c morphism at different weight c's before and after the action of the morphism $T_{a,b}$

Γ (p.926) is the Lie algebra $\mathfrak{g}$ with coefficients in the ring of regular functions over a genus-zero complex curve with finitely many points removed

γ_s (4.2) is a "chart": $P^1 \to C$, or, a morphism of varities from the complex projective line P^1 to a genus zero curve C with a unique marked point $p_s \in \heartsuit$ on every component, such that $\gamma_s(0) = p_s$



Γ (4.6, p.926) is the central extension $\Gamma = (R \otimes \mathfrak{g}) \oplus \mathbb{C}\mathbf{1}$ of the Lie algebra of the Lie-valued regular functions on the algebraic curve C'

G_N (4.8, p.927) the complex subspace of $U(\Gamma)$ spanned by products $(f_1c_1)\ldots(f_Nc_N)$ with $f_i \in R_1$ and $c_i \in \mathfrak{g}$.

$g_s(p)$ (7.2, p.938) ,for $s \in [s_0]$, $s_0 \in \heartsuit$, is the regular function in $R = \mathbb{C}[C']$ given by $g_s(p) = \frac{1}{1/\gamma_{s_0}^{-1}(p) - 1/\gamma_{s_0}^{-1}(\gamma_s(0))}$ on the s_0-component ($p \in C_{s_0}$) and $g_s = 0$ on other components ($p \in C - C_{s_0}$); These functions are introduced in the demonstration (7.3, 7.6) of the finite-dimensionality of W/G_MW, which in turn establishes the isomorphisms which state that $T(W)^{\#}$ and $T'(W)$ are dual to each other under D (Theorem 7.9, p.940). Note that after the definition at [Kazhdan-Lusztig I, p.938, 7.2(a)], the first property of the function g_s should read "${}^{s_0}g_s \in \epsilon + \epsilon^2\mathbb{C}[[\epsilon]]$".

$g(s_0)$ (7.3, p.938) is a function in R such that its expansion ${}^{s_0}g((s_0)$ at s_0 is in $\epsilon + \epsilon\mathbb{C}[[\epsilon]]$. A possible choice of $g(s_0)$ is the function $g_s(p)$ where $s \in [s_0] - \{s_0\}$

Γ_B (8.2) is the B-Lie algebra $B \otimes \Gamma$, where Γ is the Lie algebra on the algebra of regular functions on the complex curve C'. There is a natural homomorphism: $\Gamma \to \Gamma_B : x \mapsto 1 \otimes_B x$

$\mathfrak{g}'$ (11.1) $= \mathfrak{g}^{\clubsuit}$; $\clubsuit$ is a finite set

$\gamma_1, \gamma_2, \gamma_{12}, \gamma_3, \gamma_4, \gamma_{34}$ are charts on the complex curve $C = P^1 \sqcup P^1$ sending the points $\{0, 1, \infty\}$ on $\mathbb{C}$ to copies of such on the curve; e.g. $\gamma_1 : P^1 \cong C_{12} : z \mapsto \frac{1}{1-z}$; and also $\gamma_{12} : z \mapsto z$.

$\mathfrak{g}$ (p.335, 19.1) a simple Lie algebra with (a_{ij}) as Cartan matrix

$\mathscr{G}$ (remark following 26.4) the functor $\mathscr{O}_\kappa \to \mathscr{D} : V \mapsto V/G_1V$.

$\hat{\mathfrak{g}}$ (400) is defined as $\hat{\mathfrak{g}}_R$

$\hat{\gamma}$ (404) is defined as $x\hat{\beta}^{(n+1)}(m) - \hat{\beta}^{(n+1)}(xm)$

$\mathscr{G}$ (406, §30) is defined as the functor: $V \mapsto V/Q_1^{\#}V$ from $\mathscr{O}_\kappa$ to $\mathscr{D}$, the category of finite dimensional representations of the Lie algebra $\mathfrak{g}_F := \mathfrak{g} \otimes_{\mathbb{C}} F$ where κ is a ring morphism: $R \to F$ from ring R to a field F. F and G are equivalences of categories; α is a functorial isomorphism between them.

$(\check{\mathscr{G}}, \check{\beta})$ (421) a morphism from the braided category $\check{\mathscr{O}}$ to the Drinfeld category

$\mathscr{G}(\theta)$ (423) an element in $Hom(V_{a+b-\alpha_i}, V_a \otimes V_b) \otimes \check{R}$, the image of $\theta \in \theta_{i;a,b}$.

Hexagon identities (19.13) the two compatibilities of associativity with commutativity in a triple tensor prodcut. They provide solutions for the Yang-Baxter equation.

$\mathscr{H}$ (17.8, p.995) a linear map on the tensor product $\bigotimes_i V_i$ defined by the Sugawara operators: $y_1 \otimes y_2 \otimes y_3 \otimes y_4 \to y_1 \otimes y_2 \otimes (L_0 - L_{-1})(y_3) \otimes y_4 + y_1 \otimes y_2 \otimes y_3 \otimes (L_0 - L_{-1})(y_4)$; this map lifts the connection map $\nabla_{t\partial/\partial t}$ on the space $\dot{Y}/\dot{\Delta}_4\dot{Y}$ of coinvariants

H (9.1) the group $PGL_2(\mathbb{C})$ of automorphisms of the projective line $P^1 = \mathbb{C} \cup \infty$

$\mathscr{H}$ (386) Lie algebra of regular vector fields on P^1; $H = PGL_2(\mathbb{C})$ its group of automorphisms; its standard basis is given by $l_1 = t^2\frac{\partial}{\partial t}$, $l_0 = t\frac{\partial}{\partial t}$, $l_{-1} = \frac{\partial}{\partial t}$.

$\mathbf{h}, \mathbf{h}^+, \mathbf{h}'\mathbf{h}'^+$ (9.14, p.955) - form a "split induction datum" (A.1, p.1008), e.g. from (9.10(a)), we can take $\mathbf{h} = (\hat{\mathfrak{g}}_B^S)^+$, $\mathbf{h}' = \Delta_B$, $\mathbf{h}'^+ = B \otimes \mathfrak{g}$. This way we can show (proposition 9.15) that the map (9.15(a)) $(B \otimes (\bigotimes_s \mathscr{N}_s))_{\mathfrak{g}} \to Y_{\Delta_B}$ is surjective and is an isomorphism; We also have the base change behaviour $(B \otimes (\bigotimes_s \mathscr{N}_s))_{\mathfrak{g}} = B \otimes \left((\bigotimes_s \mathscr{N}_s)_{\mathfrak{g}}\right)$

$\mathscr{H}$ (17.8, p.995) a linear map on the tensor product $\bigotimes_i V_i$ defined by the Sugawara operators: $y_1 \otimes y_2 \otimes y_3 \otimes y_4 \to y_1 \otimes y_2 \otimes (L_0 - L_{-1})(y_3) \otimes y_4 + y_1 \otimes y_2 \otimes y_3 \otimes (L_0 - L_{-1})(y_4)$; this map lifts the connection map $\nabla_{t\partial/\partial t}$ on the space $\dot{Y}/\dot{\Delta}_4\dot{Y}$ of coinvariants

$H(V, W)$ (401) is defined as $Hom_{\mathscr{O}}(V, W)$. It is (lemma 29.8) a free R-module of finite rank.

$\hat{i}$ (412)

$\bar{i}$ (19.1) is the label for the corresponding element in the root system for α_i in the involution induced by the longest element w_0 of the Weyl group of $\mathfrak{g}$

I (19.1) a finite set, to be the set of indices of simple roots

I_n (398) is defined as the set $\{k \in K | kn \in \mathbb{N}\}$

$\tilde{i}$ (406) is a natural embedding: $\mathbf{1} \to M$ (corollary for lemma 29.12); We have $e \circ \tilde{i} = (x - \kappa)^{N-1}$.

$i_{\mathbf{V}_a}$ (412) is a morphism $\mathbf{1} \longrightarrow \mathbf{V}_a^{\hat{\kappa}} \dot{\otimes} D(\mathbf{V}_a^{\hat{\kappa}})$.

i_M (436) is a morphism $\mathbf{1} \to M \otimes_R M^*$

$j(w)$ (17.23, p.1003) image of an element $w \in T(W)^{\#}$ in $T(W_n)^{\#}$, w being represented as a sequence $(w_1, w_2, \dots)$ in $hatW$

j (415) is defined as $e \otimes 1 \otimes e \circ 1 \otimes i \otimes 1$,a morphism from $(\mathbf{V}_a^{\kappa})^* \dot{\otimes} \mathbf{V}_a^{\kappa} \longrightarrow T^{\kappa} \otimes T^{\kappa *}$

$\hat{j}$ (415) is a morphism: $(\mathbf{V}_a^{\hat{\kappa}})^* \otimes \mathbf{V}_a^{\hat{\kappa}} \to T \otimes T^*$

$\tilde{j}$ (415) is defined as $\hat{j} \otimes F_{\kappa}$

$\chi^t(V, a)$ (p.396) is the alternating sum $\sum_{i \in \mathbb{N}} (-1)^i \cdot dimExt^i_{\mathcal{O}^t_{\kappa}}(V, D(\mathbf{V}_a^{\kappa}))$

$l_{ss'}, m_{ss'}, k_{ss'}$ (10.10) they are elements in A; $l_{ss'}, m_{ss'}$ being the coefficients in the expansion ${}^{s'}f_s = l_{ss'} + m_{ss'}\epsilon + \cdots \in A[[\epsilon]]$, where $s \neq s'$; $k_{ss'} \in A'$ is defined by $f_s f_{s'} = l_{ss'} f_{s'} + l_{s's} f_s + k_{ss'} \mathbf{1}$. In fact, $k_{ss'}$ is constant along any fibre of $\underline{\mathscr{V}}' \to \underline{\mathscr{V}}$ and so $k_{ss'} \in A$.

ξ (10.13) is an element $\xi = \sum_{j,s} a_{j,s} \theta_{j,s} \in \mathscr{D}_2$; $a_{j,s} \in A$. The coefficients $a_{0,s}, a_{-1,s}$ satisfy a relation with the coefficients $l_{ss'}$.

$\hat{\kappa}$ (401) the natural embedding $R \to R_{\kappa}$

κ_0 (410) is an element in $\mathbb{C} - \mathbb{Q}_{\geq 0}$

κ (430) is a complex number

$\mathbf{L}_a^{\kappa}$ (2.8) irreducible quotient of $\mathbf{V}_a^{\kappa}$, where $(a, \kappa - h)$ is the highest weight of the module

L_k (1.14, 1.15) the Sugawara operator, defined by

$$L_k(x) = \frac{1}{2\kappa} \sum_{j \geq -k/2} \sum_p (\epsilon^{-j} c_p)(\epsilon^{j+k} c_p) x + \frac{1}{2\kappa} \sum_{j < -k/2} \sum_p (\epsilon^{j+k} c_p)(\epsilon^{-j} c_p) x$$

where $\{c_p\}$ is an orthonormal basis of $\mathfrak{g}$; if it is acting on a tensor product over the set $\clubsuit$, we denote the action on the t-th factor as $L_{k;t}$; and the total action is $L_k = \sum_{t \in \clubsuit} L_{k;t}$ (1.15)

${}_{\lambda}V$ (2.23, p.918) subspace of vectors annihilated by some power of $(L_0 - \lambda)$; for example, we have $D(V) = \bigoplus_{\lambda} {}^d({}_{\lambda}V)$, where ${}^dV$ is defined in (1.16)

$l_{ss'}, m_{ss'}, k_{ss'}$ (10.10) they are elements in A; $l_{ss'}, m_{ss'}$ being the coefficients in the expansion ${}^{s'}f_s = l_{ss'} + m_{ss'}\epsilon + \cdots \in A[[\epsilon]]$, where $s \neq s'$; $k_{ss'} \in A'$ is defined by $f_s f_{s'} = l_{ss'} f_{s'} + l_{s's} f_s + k_{ss'}\mathbf{1}$. In fact, $k_{ss'}$ is constant along any fibre of $\underline{\mathscr{V}}' \to \underline{\mathscr{V}}$ and so $k_{ss'} \in A$.

$\Lambda_{\theta_{k,s}}$ (11.2) where $k = 0, +1, -1; s \in S$ is defined to be $1 \otimes \cdots \otimes L_{k;t} \otimes \cdots \otimes 1$; where $L_{k;t}$ is the Sugawara opeartor L_k acting on $\underline{V}_s$ (p.953) at the s-th position in $Y = \bigotimes_{s \in S} \underline{V}_s$

Λ (11.2) is a complex-linear map: $\mathscr{D}_0 \to End_B(Y) : \partial \mapsto \Lambda_\partial$. It is actually (11.2(a)) a homomorphism of Lie algebras over $\mathbb{C}$.

Λ', Λ'' (11.3) $\Lambda' : \mathscr{D}_1 \to End_B(Y)$ is the unique A-linear extension of Λ to $\mathscr{D}_1$; Λ'' is the restriction of Λ' to $\mathscr{D}_2$. Λ'' is a homomorphism of A-Lie algebras.

$\hat{L}_0$ (26.3) is defined as $\varprojlim L_0(N)$.

$\lambda_a(n)$ (393) is defined as $\frac{1}{2\kappa} \langle a, a+2 \rangle + m$

Λ, F, M, A (405) Λ is a ring of formal power series over $\mathbb{C}$; F its field of fractions; and M a free finitely generated module over Λ with a filtration $0 = M_0 \subset M_1 \subset \cdots \subset M_N = M$ such that the quotients are torsion free; A an endomorphism of M preserving the filtration and such that for every $i : 1 \leq i \leq N$, the quotient modules $\bar{M}_i = M_i / M_{i-1}$ decomposes as $\bar{M}_i = \bigoplus_{j=1}^{n_i} M^{\alpha^i_j}$

$\lambda_{2\rho}$ (411) is defined as $\prod_{i \in I} \lambda_i^{c_\imath}$

L (415) is defined as $ker(e_{\mathbf{V}_a^\kappa}) \subset (\mathbf{V}_a^\kappa)^* \otimes \mathbf{V}_a^\kappa$

$\hat{L}$ (415) is defined as $ker(e_{\mathbf{V}_a^{\hat{\kappa}}}) \subset (\mathbf{V}_a^{\hat{\kappa}})^* \otimes \mathbf{V}_a^{\hat{\kappa}}$

$\tilde{L}$ (415) is defined as $\hat{L} \otimes F_\kappa \subset ker(\tilde{j})$

$\mathscr{L}$ (436) is an irreducible object in $\mathscr{C}_\kappa$

$l_{ss'}, m_{ss'}, k_{ss'}$ (10.10) they are elements in A; $l_{ss'}, m_{ss'}$ being the coefficients in the expansion ${}^{s'}f_s = l_{ss'} + m_{ss'}\epsilon + \cdots \in A[[\epsilon]]$, where $s \neq s'$; $k_{ss'} \in A'$ is defined by $f_s f_{s'} = l_{ss'} f_{s'} + l_{s's} f_s + k_{ss'}\mathbf{1}$. In fact, $k_{ss'}$ is constant along any fibre of $\underline{\mathscr{V}}' \to \underline{\mathscr{V}}$ and so $k_{ss'} \in A$.

$\mathscr{M}$, $\dot{\mathscr{M}}$ (17.10) $\mathscr{M} = Y_\infty/\Delta_{4,\infty}Y_\infty$, $\dot{\mathscr{M}} = \dot{Y}/\Delta_4\dot{Y}$. It is an A_∞-module and has a natural connection with regular singularity at 0 (17.14, p.998).

$\tau(M)$ (17.24, p.1004) torsion module of M over A_∞, i.e. vectors annihilated by powers of t.

M^f (17.24) $= M/\tau(M)$

$\hat{M}$ (17.24) $= \varprojlim_{n\geq 1} M/t^n M$

$\hat{\mathbf{M}}$ (17.24, p.1005) $= Hom_{\mathbb{C}[[t]]}\left(\hat{\mathscr{M}}'^f, \hat{\mathscr{M}}^f\right)$

$\mathscr{M}_{an}, \mathscr{M}'_{an}, \mathbf{M}_{an}$ (17.26, p.1005) real analytic bundles on the real interval $(-\infty, 1)$

$M(K)$ (398) is defined as $M \otimes_R K$

${}_\Lambda M^\perp$ (399) is defined as $\bigoplus_{\substack{\Lambda' \in T(A,M) \\ \Lambda \neq \Lambda'}} {}_{\Lambda'}M$.

$M_i^{(n)}$ (403) is defined as $M_i/(x-\kappa)^n M_i$ where i is equal to 1 or 2.

$\check{M}$ (404) image in $\mathscr{O}_{\check{\kappa}}$ of any object M in $\mathscr{O}_{\hat{\kappa}}$

$m^Z_{(i,j)}$ (405), where Z is a subset of Q, is an endomorphism of $\check{M}$, defined as follows: $\alpha^i_j - \check{B}$ for $(i,j) \notin Z$; and $-\check{C}$ for $(i,j) \in Z$.

M (407) finitely generated torsion-free R_∞-module

M_{an} (407) is defined as $M \otimes_{R_\infty} A_{an}$

m_0 (418) a constant

$\check{m}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

$M_{V,V'}$ (426)

$\bar{M}_{W,W'}$ (428) the unique functorial isomorphism $\bar{X}(W) \otimes \bar{X}(W') \to \bar{X}(W \dot{\otimes} W')$ in $\mathscr{O}_\kappa$, satisfying a commutation relation between the M and r morphisms. (429)

$\mathcal{N}$ (2.1) nil-module, that is, a finite-dimensional module over the $\mathbb{C}$-algebra $\mathbb{C}[\epsilon]\otimes\mathfrak{g}$, and the ideal $\epsilon\mathbb{C}[\epsilon]\otimes\mathfrak{g}$ acts nilpotently on $\mathcal{N}$, i.e., there exists a fixed number $t>1$ such that any t elements in $\epsilon\mathbb{C}[\epsilon]\otimes\mathfrak{g}$ acts on $\mathcal{N}$ as zero

$\mathcal{N}^\kappa$ (2.3) generalized Weyl module induced by the nil-module $\mathcal{N}$

$M(\kappa)$ (section 29, p.397) is the quotient $M/(x-\kappa)$

$\mathbb{N}$ in the paper, the set of natural numbers is defined as the set of non-negative integers, including zero (?)

ν (4.16,p.931) is a $\mathbb{C}$-linear map: $T(W)\otimes X \to (W\otimes X)/G_1(W\otimes X)$, defined canonically; the image $\nu(\tilde{\mathfrak{g}}^\heartsuit(T(W)\otimes X))$ is contained in $\Gamma(W\otimes X)/G_1(W\otimes X)$ (4.17,p.932); Thus we have [1] an induced canonical map of coinvariants $(T(W)\otimes X)_{\tilde{\mathfrak{g}}^\heartsuit}\to(W\otimes X)_\Gamma$.

∇ (12.2, p.967) a connection on $\mathscr{A}$-module M, i.e. a $\mathbb{C}$-linear map $\nabla: M\to \Omega^1_{\mathscr{A}}\otimes_{\mathscr{A}}M$, satisfying the Leibniz condition $\nabla(am)=d(a)\otimes m+a\nabla(m)$ for $a\in\mathscr{A}, m\in M$; where $\Omega^1_{\mathscr{A}}=\mathscr{I}/\mathscr{I}^2$ is an $\mathscr{A}$-module and $\mathscr{I}$ is the kernal of the multiplication map $\mathscr{A}\otimes\mathscr{A}\to\mathscr{A}$. There is an isomorphism $Hom_{\mathscr{A}}(\Omega^1_{\mathscr{A}},\mathscr{A})\cong Der(\mathscr{A})$ given by $f\mapsto fd$.

∇_∂ (12.3) is a map in $End_{\mathbb{C}}(M)$; given a derivation $\partial\in Der(\mathscr{A})$, ∇_∂ is given by the composition of ∇ with $\partial\otimes 1_M$:

$$M\to\Omega^1_{\mathscr{A}}\otimes_{\mathscr{A}}M\to\mathscr{A}\otimes_{\mathscr{A}}\otimes M\cong M$$

$\nabla_\partial(m)$ is $\mathscr{A}$ linear in the two variables m and ∂.

∇^0,∇^1 (12.7, p.968) they are connections on the A-module Y, with $\nabla^0_\partial(fy)=\partial(f)y$ and $\nabla^1_\partial=\nabla^0\pi(\partial)+\Lambda'_\partial$, where $\pi:\mathscr{D}_1\to Der A$ is the canonical map. $\nabla^1:\mathscr{D}_1\to End_{\mathbb{C}}(Y)$ is a homomorphism of Lie algebras over $\mathbb{C}$.

∇,∇' (17.16) are $\mathbb{C}$-linear maps: $\mathbb{C}[t]\otimes X_{12}\otimes X_{34}\to\mathbb{C}[t]\otimes X_{12}\otimes X_{34}$ defined by the Sugawara operator $L(0)$. They induce the same complex-linear map $\nabla_{t\partial/\partial t}$ on the space of coinvariants $(\mathbb{C}[t]\otimes X_{12}\otimes X_{34})_{\Delta_{2,\infty}}$, and on this $\mathbb{C}[t]$-module it defines a connection with regular singularity

[1]Γ being the Lie algebra $\mathfrak{g}$ tensored with the ring of regular functions over a complex curve, see p.926

ν (17.1) a morphism: $\dot{C} \to \underline{\mathscr{V}}$

$[n]_i$ (19.1) $= \frac{v_i^n - v_i^{-n}}{v_i - v_i^{-1}}$

$\tilde{\nabla}$ (p.387) is defined as $\nabla_0 + \hat{\omega}$.

∇^{tr} (p.387) is the trivial connection on $A_\infty \otimes \bar{Y}/\mathfrak{g}\bar{Y}$

$\tilde{\mathscr{N}}$ (392) the U^κ-module induced from the $U(\mathfrak{g})$-module $\mathscr{N}$.

$\mathbb{N}_0$ (393) a subset of $\mathbb{N}^I$, depending on the rationality of κ

$\tilde{\mathscr{N}}$ (401) the object in $\mathscr{O}_\varphi$ corresponding to the nilmodule $\mathscr{N}$.

$\alpha^{(n)}$ (403) is a morphism $M \to M_2^{(n)}$ of $\hat{\mathfrak{g}}_{R_\kappa}$-modules (prop.29.3)

N_j (405)

$\nabla^0_{t\partial/\partial t}$ denotes the trivial connection on M_{an}

$\nabla_{t\partial/\partial t}$ (407) a connection on $\mathscr{M}_{an}$

$\nabla'_{t\partial/\partial t}$ (407) a connection on $\mathscr{M}_{an}$

$(\mathscr{N}, \nabla_{\partial/\partial t})$ (408) an R-vector bundle over $\mathbb{C} - \langle 0, 1\rangle$ with connection $\nabla_{\partial/\partial t}$ being (the trivial connection) + $-\frac{1}{\kappa}(\frac{\Omega_{12}}{t} - \frac{\Omega_{23}}{1-t})dt$

$\langle \mathscr{N} \rangle$ (408) space of flat sections $n(t)$ of $\tilde{\mathscr{N}}$ over the interval $(0, 1)$.

$n_a(\kappa)$ (412) the order of zero of d_a at κ

$\mathbb{N}^I(\theta)$ is defined as $\left\{a \in \mathbb{N}^I \middle| \ \theta_a|_{Z_\theta} \equiv 1\right\}$

$\tilde{\mathbf{N}}$ (417) is defined as $\{a \in \mathbf{N}^I(\theta) | T_a^\kappa$ is projective$\}$

$\bar{\mathbf{N}}$ (417) is defined as the image of $\tilde{\mathbf{N}}$ in $\mathbf{N}^I(\theta)/p\mathbf{N}^I(\theta)$

$\mathbf{N}^I(m)$ (418) a subset

$[n]$ (433) is defined as $\frac{v^n - v^{-n}}{v - v^{-1}}$ for an integer n

$\{\omega_i | \ i \in I\}$ (19.1) the collection of simple roots

$\mathcal{O}_\kappa$ (Kazhdan and Lusztig's introduction, p.905) - "category O", consisting of modules of finite length over $\hat{\mathfrak{g}}$, with central charge $\kappa - h$, and whose composition factors are irreducible highest weight modules corresponding to weights which are dominant in the $\mathfrak{g}$ -direction (p.905)

$\mathcal{O}_\kappa^t$ (3.3) a full subcategory of $\mathcal{O}_\kappa$ whose objects have composition factors $\mathbf{L}_a^\kappa$ for some a in the set F^t; it is closed under extension and duality $V \to D(V)$

$\langle \omega c \rangle$ (4.9, p.927) $\hat{\mathfrak{g}}$ module structure on the completion $\hat{W}$ defined through the approximation property 4.9(a) of the space R of regular functions on C'

ω (6.3,p.934) a collection $(\omega_s)_{s \in \heartsuit} \in \mathbb{C}((\epsilon))^\heartsuit$ of Laurent series ω_s, such that $\omega c \in \hat{\mathfrak{g}}^\heartsuit$ acts on Z^∞ via $\langle \omega c \rangle \lambda = (gc)\lambda$ where $g \in R = \mathbb{C}[C']$ is a regular function on the curve C' which expands, at each point $s \in \heartsuit$, to a series approximating ω to the N-th order, i.e. ${}^s g - \omega_s \in \epsilon^N \mathbb{C}[[\epsilon]]$ for every $s \in \heartsuit$.

$\Omega^1_{\mathscr{A}}$ (967, section 12) is the space of 1-forms over a $\mathbb{C}$-algebra $\mathscr{A}$, defined algebraically as $\mathscr{I}/\mathscr{I}^2$, where $\mathscr{I}$ is the kernel of the multiplication map $\mathscr{A}^{\otimes 2} \to \mathscr{A} : a \otimes b \mapsto ab$. $\Omega^1_{\mathscr{A}}$ is an $\mathscr{A}$-module (not just a $\mathbb{C}$-module) since it is a quotient by $\mathscr{I}^2$. Given an $\mathscr{A}$-module M, a connection over M is defined as a $\mathbb{C}$-linear map $\nabla : M \to \Omega^1_{\mathscr{A}} \otimes M$ such that $\nabla(am) = d(a) \otimes m + a\nabla(m)$, where $d(a) = a \otimes 1 - 1 \otimes a$. If f is a homomorphism from $\Omega^1_{\mathscr{A}} \to \mathscr{A}$, then the map $f \mapsto fd$ defines an isomorphism $Hom_{\mathscr{A}}(\Omega^1_{\mathscr{A}}, \mathscr{A}) \to Der(\mathscr{A})$

ω_n (17.27(a)) an element in $\mathbf{M}/t^n\mathbf{M}$ constructed from an isomorphism of quotients of $\mathscr{M}$ and $\mathscr{M}'$.

$\tilde{\omega}$ (17.27, 17.28) an analytic section of $\mathbf{M}_{an}$, horizontal with respect to the connection $\nabla_{t\partial/\partial t}$, which is the analytic continuation along the interval $(-\infty, 1)$ of the power series ω at 0,

ω_i (19.1) fundamental weights

$\Omega^{(2)}$ (26.3) is defined as $\sum_p c_p \otimes c_p \in U(\mathfrak{g}) \otimes U(\mathfrak{g})$

$\hat{\omega}$ (p.387) is defined as $-\frac{1}{\kappa}\left(\sum_{i=1}^{n}\Omega_i\omega_{i,0}+\sum_{i=1}^{n}f_{i,j}\Omega_{i,j}^{(2)}\omega_{i,-1}\right)$. Since it commutes with the action of $\mathfrak{g}$ on $\hat{Y}$, it defines a connection Δ on the bundle $A\otimes\bar{Y}/\mathfrak{g}\bar{Y}\cong Y/\Delta Y$. It coincides (cor. 26.1) with the connection defined in §12.11.

$\mathscr{O}_\kappa(z)$ (393) a certain full subcategory of $\mathscr{O}_\kappa$ where all irreducible subquotients of V are isomorphic to L_a^κ for some $a\in z\cap\mathbb{N}^I$; where $z\in Z$ is a W_κ-orbit on $\mathbb{Z}^I$.

$\mathscr{O}_\kappa^a, \mathscr{O}_\kappa^{<a}$ (396) are full subcategories of $\mathscr{O}_\kappa$ with composition factors of some given form, their highest weights being restricted to some subsets

$\mathscr{O}$ (400) the full subcategory of $\tilde{\mathscr{O}}$ of $\hat{\mathfrak{g}}$ modules, consisting of those which are free as R-modules, and for any $\kappa\in S$, the module $V(\kappa)$ belongs to the category $\mathscr{O}_\kappa$.

$\mathscr{O}$ (401) is the category of $\hat{\mathfrak{g}}_{\tilde{R}}$-modules defined analogously to $\mathscr{O}$

$\mathscr{O}_\infty$ (401) is defined as $\mathscr{O}_{\kappa_\infty}$

$\mathscr{O}^a$ (402) the full subcategory of $\mathscr{O}$, consisting of modules V such that, for any $\kappa\in S$, $V(\kappa)$ lies in $\mathscr{O}_\kappa^a$. (§29.8, §28)

$(\mathscr{O}_\infty,\dot{\otimes},a,\mathscr{P})$ (410) is a braided category (corollary, p.410); $(\mathscr{G},\beta)$ defines an equivalence between $\mathscr{O}_\infty$ and $\mathscr{D}$ (corollary, p.410); where $\beta:\mathscr{G}(\tilde{W}_1)\otimes\mathscr{G}(\tilde{W}_2)\to\mathscr{G}(\tilde{W}_1\dot{\otimes}\tilde{W}_2)$ (lemma 30.2, p.407)

Ω_κ (412) the intersection of the W_κ-orbit of 0 with with $\mathbb{N}^I$.

$\mathscr{O}_\kappa^\theta$ is a subcategory of $\mathscr{O}_\kappa$, consisting of modules whose irreducible subquotients are isomorphic to L_a where $a\in\mathbb{N}^I(\theta)$

Pentagon identity (19.11) compatibility of associativity isomorphism with itself in a quadruple tensor product

ϕ (10.3) is a map: $H\times P^1\to P^1$ given by the evaluation $\phi(\gamma,z)=\gamma(z)$. The vector fields θ_k act on the first factor γ, and we have $\theta_k(\phi)=z^{k+1}\frac{\partial\phi}{\partial z}$

$p_{n,s'}$ (10.8) n-th coefficient of the series expansion of a function $f\in A'$ at a point $\gamma_{s'}(0)$

$P, \bar{P}$ (14.2) are maps: $V \otimes V' \otimes V'' \to V' \otimes V \otimes V''$ given by $P(x \otimes y \otimes z) = \tau y \otimes \tau x \otimes \bar{\tau} z$ and $\bar{P}(x \otimes y \otimes z) = \bar{\tau} y \otimes \bar{\tau} x \otimes \tau z$. There are induced isomorphisms of coinvariants (14.5) $(V \otimes V' \otimes V'')_{\Delta_{\mathbb{C}}} \to (V' \otimes V \otimes V'')_{\Delta_{\mathbb{C}}}$ where $\mathbb{C}$ is an A-algebra via evaluation at a point.

$\mathscr{P}, \bar{\mathscr{P}}$ (14.6(d)) is the transpose $(V \dot{\otimes} V') \to (V' \dot{\otimes} V)$ in $\mathscr{O}_\kappa$ of the map $D(V' \dot{\otimes} V) \cong D(V \dot{\otimes} V')$ constructed at (14.6(b)). The two maps $\mathscr{P}, \bar{\mathscr{P}}$ are in fact inverse of each other (p.977).

Φ_n (section 15, p.977; 15.24, p.983; 15.25) where $n \geq 1$, is a family of maps relating the two spaces of coinvariants on non-singuar and degenerate quadrics (i.e. union of two lines). Φ_n is constructed as a B_n-linear map (15.25(a), p.984): $T(W_n)^{\#}/\Delta_{2,n} T(W_n)^{\#} \to W_n/\Delta_{4,n} W_n$, where $1 \leq n < \infty$. It is shown in §16 that Φ_n is an isomorphism for every $n \geq 1$. (proposition 15.27, p.985)

Ψ_n (15.20, p.982) is an isomorphism of B_n-modules (15.20(a)):

$$\left(B_n \otimes (V_1 \dot{\otimes} V_2) \otimes (V_3 \dot{\otimes} V_4)\right)_{\Delta_{2,n}} \cong T(W_n)^{\#}_{\Delta_{2,n}}$$

$P_0, P, \hat{P}_0, \hat{P}$ (16.9) are Lie algebras with coefficients being rational functions or formal power series; e.g. $P = P_0 + B_n \mathbf{1}$ where $P_0 \cong \frac{B_n[p,q]}{pq-t} \otimes \mathfrak{g} \subset \tilde{\mathfrak{g}}^{\heartsuit}_{B_n}$

Π_n (16.17, p.992; 17.23,p.1003) (see 8.5(a), p.994) a map from $\tilde{\mathfrak{g}}^{\heartsuit}_{B_n}$-coinvarinats to Γ-coinvariants. It is compatible (17.23) with the connections $\nabla_{t\partial/\partial t}$.

ϖ (§19) a formal variable, to be thought of as $1/\kappa$.

Π_0, Π_1 (19.8, 19.9, 19.10) endomorphisms of V, in particular, the quadratic commuting element t acting on the 12- or 23- components of $V_1 \otimes V_2 \otimes V_3$.

π (26.4) is defined as the natural projection $Y \to Y/\Delta Y$

$\varphi(t)$ (387) is defined as $-\frac{q}{\kappa}\left(-\frac{t}{1-t}\Omega^{(2)}_{13} + \Omega_{34}\right)$. Its sum with the trivial connection ∇^{tr} is equal to (Lem.26.6) $\nabla_{t\partial/\partial t}$

φ_t (387) is defined as the map: $\bar{\mathscr{M}}(0) \to \mathscr{M}(0)$ defined in §15.25, p.984 (???), which can be considered as an endomorphism on $\bar{Y}/\mathfrak{g}\bar{Y}$.

$\mathscr{P}^{\kappa}_a(\lambda)$ (388) is defined as $U(\hat{\mathfrak{g}}^-) \otimes_{U(\mathfrak{g})} \mathscr{V}_a$

φ (401) is some ring homomorphism $R \to \tilde{R}$

$\mathscr{P}$ (407, §31) commutativity isomorphism for $(V_1 \otimes V_2)$

φ (407, §31) isomorphism $\left\langle V_1 \dot{\otimes} V_2, V_3 \dot{\otimes} V_4 \right\rangle \tilde{\to} \langle V_1, V_2, V_3, V_4 \rangle$ defined as in §17.29 (p.1008), for objects $V's$ in $\mathscr{A}$.

$\hat{P}$ (407, §30) commutativity isomorphism for fusion tensor products of objects in $\mathscr{O}_\infty$.

φ_a (411) is defined as the composition ... ,which is a map V_a^∞ to itself. It is part of the "rigidity" structure.

P (412) is the root lattice

P^κ (416) projective module covering the irreducible quotient $\mathbf{L}_b^\kappa$.

φ^- (416) an involution of D

φ (421) an embedding $\mathscr{V}_{a+b} \to \mathscr{V}_a \otimes \mathscr{V}_b$.

$\check{\varphi}^e_{i;a,b}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

$\check{\psi}^e_{i;a,b}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

$\check{\Phi}^c_{i;a,b}$ (426)

$\check{\Psi}^c_{i;a,b}$ (427)

φ_c (429)

φ (431) is a pairing between $\mathbf{X}_c$ and $\mathbf{X}_c^*$ defined via the morphisms M and $\check{s}_c$.

ψ (432)

$\hat{\psi}$ (432)

P (433) is a polynomial in x,y, with coefficients in the ring of rational functions in $\mathbb{C}$.

ϕ (435) is a bilinear pairing of X with values in $\frac{1}{c}\mathbb{Z}$

Pi_ϕ (435) is defined as $(m \otimes m') \mapsto v^{\phi(\lambda,\lambda')} m \otimes m'$ where the lambda's are weights of the elements m's

P^κ_γ (438)

Q_k (1.7) is the A-submodule of $U(\tilde{\mathfrak{g}}_A)$ generated by products $(\epsilon c_1)\dots(\epsilon c_k)$, where $c_k \in \mathfrak{g}$

$q_{n,s'}$ (10.8) n-th coefficient of the series expansion of $\theta_{k,s} f$ at a point $\gamma_{s'}(0)$.

Q_N (1.7, p.908)

Q (16.11) a Lie subalgebra of $\tilde{\mathfrak{g}}^{\heartsuit}_{B_n}$, in which it is the complement of P.

Q (405) is defined as the set of pairs (i,j) such that $1 \le i \le N$, $1 \le j \le n_i$, and $(i,j) \neq (1,1)$.

Q_0 (405)

Q^κ_γ (438)

$r_{a,b}$ (10.16(a)): coefficients for the change of basis from $\Theta_{a-b,s}$ to σ_k induced by the isomorphism of the space $H^S \simeq H^S$ given by (γ_s)

R (4.5, p.926) is the algebra of regular complex-valued functions on the algebraic curve C' obtained from removing points $\{p_s | s \in S(= \spadesuit \sqcup \heartsuit)\}$from the genus-zero curve C

R_n (4.8, p.927) is a subspace of $R = \mathbb{C}[C']$, consisting of those functions which have an n-th order zero (or (-n)-th order pole if n is negative) at every point $s_0 \in \heartsuit$.

$r_{s,s'}$ (Lemma 11.9)

R (15.18, see also 4.5, p.926) algebra of regular functions on the curve with points removed; the case of §15.18, where the curve $C = P^1 \sqcup P^1$ and $S = \{1,2,12,3,4,34\}$, we have $R = \mathbb{C}[u, u^{-1}, (1-u)^{-1}] \oplus \mathbb{C}[v, v^{-1}, (1-v)^{-1}]$

$R(\Pi_0, \Pi_1)$ (p.339) is the isomorphism $R_1 \cdot R_0^{-1} : V \to \mathscr{X} \to V$

R (397) the ring of analytic functions over an unbounded open subset $S \subset \mathbb{C}$ meromorphic at infinity

R_κ (398) is defined as the completion of R at κ

R (400) is defined as R_S = the ring of analytic functions on S, meromorphic at infinity (p.397)

$r(\kappa)$ (401) is a natural embedding $H(V,W)(\kappa) \to Hom_{\mathscr{O}_\kappa}(V(\kappa), W(\kappa))$. (lem.29.8); tensoring with R_κ, we have an isomorphism $r(\hat{\kappa}) : H(V,W) \otimes_R R_\kappa \tilde{\to} Hom_{\mathscr{O}_{\hat{\kappa}}}(V \otimes_R R_\kappa, W \otimes_R R_\kappa)$.

R_∞ (407, §31) completion of R at infinity

ρ (411) is the sum of fundamental weights

r (421) a positive rational number, depending on the type of the root system

${}_iR$ (435) is a $\mathbb{C}(v)$-linear map of $\mathbf{f}$

$\mathscr{R}_{M',M}$ (435) is defined as $\Theta \circ \Pi_\phi \circ \mathbf{s}$

stable object (432) a V in $\mathscr{O}_\kappa$ such that for every V', the map $M_{V,V'}$ of $\bar{\mathbf{X}}(\_)$'s is an isomorphism

S_c (20.12): the morphism S'_c normalised by an "admissible collection" (20.11) of coefficients $\{g_a\}$ satisfying some cocycle conditions with the set $\{g_{a,b}\}$ (20.6(a))

S'_c (20.4): a morphism: $V_{\bar{c}} \otimes V_c \to V_0 \simeq A$ = the ring of formal power series over a variable (which will turn out to be $1/(k+\check{h})$, the level plus the dual coxeter number), whose value at the highest weight vector $x_{\bar{c}} \otimes y_c$ is normalised as 1

s_i (19.1) simple reflections

S (9.1) a finite set of at least two elements

σ_k (10.14) where $k = 0, +1, -1$, is a vector field on H^S obtained from θ_{k,s_0} through left translation by $\gamma_{s_0}^{-1}$ for every H-factor for which $s \neq s_0$, where s_0 has been fixed from the beginnning. We have $\sigma_k \in \mathscr{D}_2$. They form an A-basis for $\mathscr{D}_2$.

S_a^κ (27.6, p.392) is defined as $Hom_{\mathscr{O}_\kappa}(\mathbf{V}_a^\kappa, D((V)_{\bar{a}}^\kappa))$. It can be considered as a bilinear form $s : \mathscr{V}_a^\kappa \times \mathscr{V}_{\bar{a}}^\kappa \to \mathbb{C}$ (p.393).

S (400) the open set $\mathbb{C} - \mathbb{R}_{\geq 0} \subset \mathbb{C}$

$\hat{s}_a^{\hat{\kappa}}$ (406) is defined as $\hat{s}_a \otimes 1$

$\check{s}_a^{\hat{\kappa}}$ (406) is an isomorphism: $\mathbf{V}_{\bar{a}}^{\hat{\kappa}} \to D(\mathbf{V}_{\bar{a}}^{\hat{\kappa}})$ defined by $\hat{s}_a^{\hat{\kappa}}$ when $\mathbf{V}_a^{\hat{\kappa}}$ is irreducible.

$\check{s}_a^{\hat{\kappa}}$ (412) is an isomorphism $\mathbf{V}_{\bar{a}}^{\hat{\kappa}} \xrightarrow{\sim} D(\mathbf{V}_a^{\hat{\kappa}})$.

$\mathscr{S}_a$ (420) is defined as $Hom_{\mathscr{O}}(V_{\bar{a}}, D(V_a)) = Hom_{\mathscr{O}}(V_{\bar{a}} \dot{\otimes} V_a, \mathbf{1})$

$\check{s}'_a$ (421) a generator of the free R-module $\mathscr{S}_a$; its choice is fixed in corollary to lemma 33.5.

$\check{s}_a$ (422) choice of $\check{s}'_a$, such that all $g_{a,b} \equiv 1$.

s_a (425) a choice of generators in $\mathscr{S}_a$

$s_a(\kappa)$ (429)

$\mathbf{s}$ (435) is the switching map for a tensor product of two modules

tilting module (p.390) is a module in $\mathcal{O}_\kappa$ which is in $\mathcal{A}_\kappa$ (i.e. has a Weyl filtration) and so does its dual $D(M)$

$T_{a,b}$ (20.1): a morphism $V_{a+b} \to V_a \otimes V_b$ of modules over $\mathfrak{g}$

t (19.2(c)): $t = (1/2)(\Delta(\Omega) - 1 \otimes \Omega - \Omega \otimes 1)$, a bilinear operator on $\mathfrak{g}$ -modules which commutes with the action of $\mathfrak{g}$

$T(W)^{\#}$ (sections 4.4,4.11, p.926, p.930) a construction of the fusion tensor product as a $\tilde{\mathfrak{g}}^{\heartsuit}$ module, from a tensor product $W = \bigotimes_{s \in \spadesuit} V_S$ where V_s is a $\mathfrak{g}$ module and $S = \heartsuit \sqcup \spadesuit$

$T'(W)$ (6.3, p.934) is the restriction of Z^∞ as a $\hat{\mathfrak{g}}^{\heartsuit}$-module to a $\tilde{\mathfrak{g}}^{\heartsuit}$-module; there are a $\tilde{\mathfrak{g}}^{\heartsuit}$ dualizing homomorphisms (6.6) $T(W)^{\#} \to D(T'(W))$ and $T'(W) \to T(W)^{\#}$ which will turn out to be isomorphisms (Remark 6.8, Theorem 7.9) under some finiteness conditions (Corollary 7.5)

$\theta_0, \theta_{-1}, \theta_1$ (10.2- , pp.957-) vector fields on $GL_2(\mathbb{C})$, i.e. derivations of the algebra $\mathbb{C}[GL_2]$, which preserve the subalgebra $A_0 = \mathbb{C}[PGL_2]$ and hence can be regarded as vector fields on $H = PGL_2$.

$\theta_{k,s}$ (10.4, p.958,957) vector fields on $\mathscr{V} \subset H^S$ which acts acts in the s-direction by θ_k ; $k = 0, +1, -1; s \in S$, forming

1. an A_0-basis of $Der(A_0)$ (10.2);
2. a $\mathbb{C}$-basis of the Lie algebra of derivations of A_0 commuting with the automorphisms of A_0 induced by left translations in H (10.2);
3. a $\mathbb{C}$-basis of $\mathscr{D}_0$, (Lemma 10.6)
4. an $\tilde{A}(=\mathbb{C}[\mathscr{V}])$-basis of $Der(\tilde{A})$, (Lemma 10.6)
5. an $A(=\mathbb{C}[\underline{\mathscr{V}}])$-basis of $\mathscr{D}_1$ (Lemma 10.6)

These operators act on $Y = \bigotimes_{s\in S} \underline{V}_s$ via the maps $\Lambda'$and Λ'' from $\mathscr{D}_1, \mathscr{D}_2$ to $End_B(Y)$ (§11.2, p.962) induced by the map Λ taking $\theta_{k,s}$ to Sugawara operators $\Lambda_{\theta_{k,s}} = 1 \otimes \cdots \otimes L_{k;t} \otimes \cdots 1$

$\Theta_{k,s,\sigma}$ (11.5) is a B-linear map: $Y \to Y$ defined as

$$1 \otimes 1 \cdots \otimes \underbrace{\left(\sum_n \theta_{k,s}(p_{n,\sigma})\epsilon^n \hat{c} \right)}_{\sigma-factor} \otimes \ldots 1 \otimes 1$$

$\tau, \bar{\tau}$ (14.1, p.974) which form parts of the commutativity isomorphisms in the category, are maps $V \to V$, given by $\mathscr{O}_\kappa$ given by $\tau = e^{i\pi L_0}e^{L_1}; \bar{\tau} = e^{-i\pi L_0}e^{L_1}$

$T(f,c), T'(f,c)$ (lemma 15.23) with $f \in \mathbb{C}[u,(1-u)^{-1}$ and $c \in \mathfrak{g}$, $T(f,c)$ and $T'(f,c)$ are two B_n-linear maps: $W_n \to W_n$

t, p, q, u, v (16.1) are variables

T (16.2, p.985) is the subalgebra $B_n[u^{-1}] \oplus B_n[v^{-1}]v^{-1}$ of the algebra $B[u,u^{-1}] \oplus B[v,v^{-1}]$.

$\tau(M)$ (17.24, p.1004) torsion module of M over A_∞, i.e. vectors annihilated by powers of t.

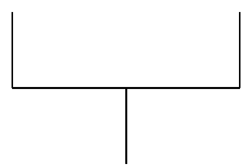
t (19.2) is an element in $\mathfrak{g} \otimes \mathfrak{g}$ which commutes with every Lie-like element $(1 \otimes x + x \otimes 1)$; Under the multiplication map it is mapped into the quadratic Casimir element $\Omega \in \mathfrak{A}$; we have $t = (1/2)(\Delta(\Omega) - 1 \otimes \Omega - \Omega \otimes 1)$

θ, θ_i (19.4) is a map: $V \to V$ defined, for $x \in V^\lambda$, as

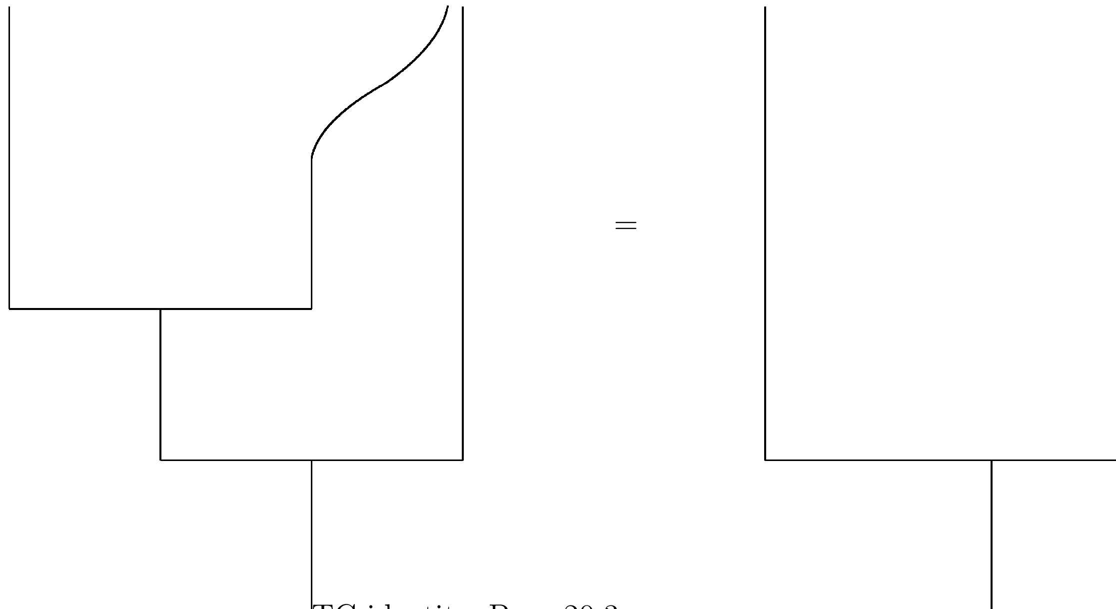
$$\theta_i(x) = \sum_{p,q,r \in \mathbb{N}; p-q+r=\lambda(i)} \frac{(-1)^q}{p!q!r!} f_i^p e_i^q f_i^r x$$

Given a choice of reduced expression of $w_0 = s_{i_1} \dots s_{i_N}$, we can define the operator $\theta = \theta_{i_1} \dots \theta_{i_N}$ which is an isomorphism of V, independent of the choice of reduced expressions. We also have $\theta(f_i x) = -e_{\bar{i}} \theta(x)$. θ acts on a tensor product as $\theta \otimes \theta$.

$T_{a,b}$ (20.1): the unique morphism $V_{a+b} \to V_a \otimes V_b$ of modules over $\mathfrak{g}$, mapping the highest weight vector to the highest weight vector

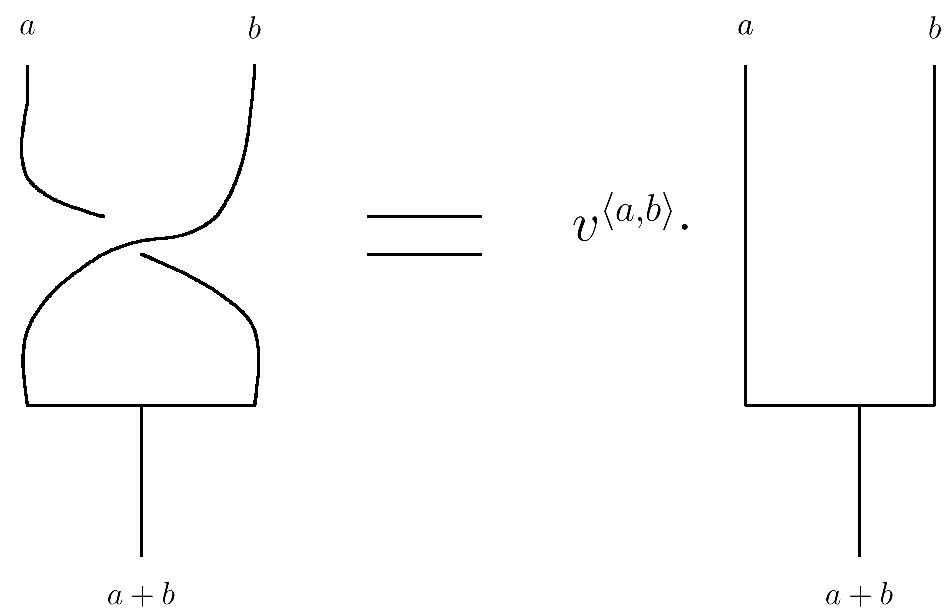


TT**-identity** (20.2) the associativity of the morphism $T_{a,b}$ with itself in an iteration

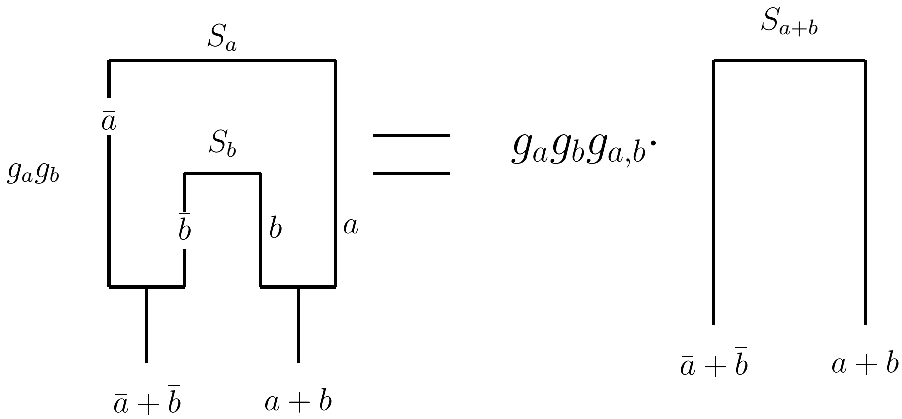


TC-identity, Prop.20.3

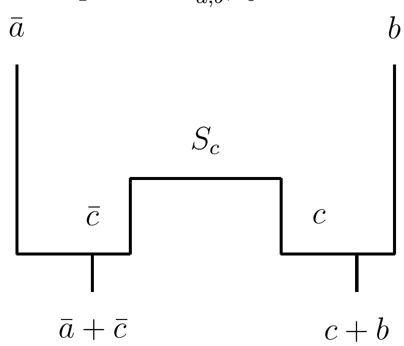
TC**-identity** (20.3) the compatibility between the T morphism of tensor products $V_{a+b} \to V_a \otimes V_b$ and the commutativity isomorphism C_{V_1,V_2}.



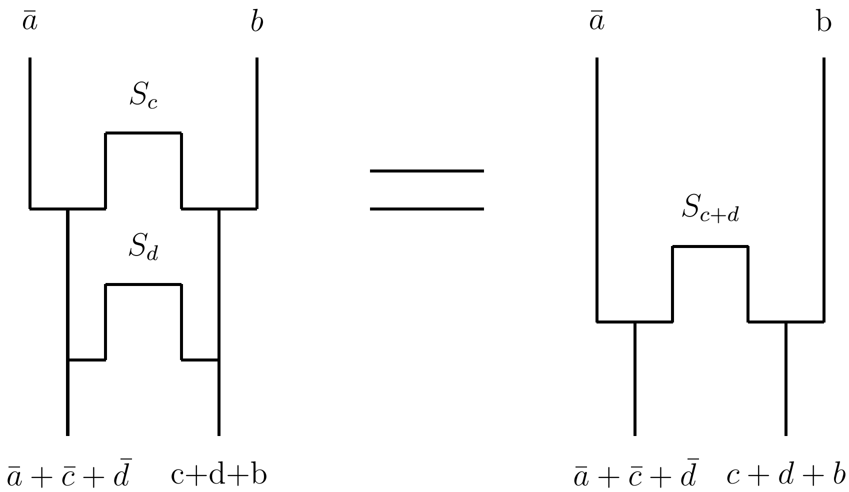
TS-identity, Prop.20.13



Morphism $tr^c_{a,b}$, §21.1



Transitivity of tr, Prop.21.3



T_0 (26.5) is defined as the submodule of $\hat{L_O}$-finite vectors in $\hat{W}$.

θ_1, θ_2 (407) are two functors from $\mathscr{O}_\kappa \times \mathscr{O}_\kappa$ to $\mathscr{D}$, taking, respectively, a pair of objects to the tensor product of its target by the functor $\mathscr{G}$ and to the target of the fusion tensor product of the pair.

τ (411) is defined as $\lambda_{2\rho}(v)$, an R_∞-point in G; where $v = e^{-\pi i/\kappa}$

T (415) lifting (prop.29.2) of T^κ to category $\mathscr{O}_{\hat{\kappa}}$

θ (415) is defined as θ_c which is the character of $Z \in G$ (§31, p.411): $\rho_c(z) = \theta_c(z) \cdot Id$

$\mathscr{T}_{a,b}$ (420) is defined as $Hom_{\mathscr{O}}(\mathbf{V}_{a+b}, \mathbf{V}_a \dot{\otimes} \mathbf{V}_b)$

$\check{T}$ (420) a generator of $\mathscr{T}_{a,b}$

$t_{a,b}$ (421) a fixed element in $\mathscr{T}_{a,b}$

$\theta_{i;a,b}$ (423) is defined as $Hom_{\check{\mathscr{O}}}(V_{a+b-\alpha_i}, V_a \otimes V_b) \otimes \check{R}$

$\tau_{i;a,b}$ an element in $Hom(V_{a+b-\alpha_i}, V_a \otimes V_b) \otimes \check{R}$, defined as a quotient of τ' by Gamma functions

$\check{\tau}_{i;a,b}$ (423) an element in $\theta_{i;a,b}$, mapping to τ (p.423), it exists by prop. 33.1

$\check{tr}^c_{a,b}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

T_ν (435) is an element in ${}_A\mathbf{f}_\nu \otimes_A ({}_A\mathbf{f}_\nu)$, defined with the dual bases b, b^* of $\mathbf{f}_\nu$.

Θ (435) is a morphism for the tensor product of two modules, defined with the maps T_ν

vector field (10.1, p.956) on a smooth variety X is an element in the Lie algebra $Der\mathbb{C}[X]$.

V_a (19.3) highest weight module of $\mathfrak{g}$

$\mathscr{V}_a$ (§1.2, p.907) irreducible representation of the simple Lie algebra $\mathfrak{g}$ with hightest weight a.s

$V(\infty)$ (section 1.9) the collection of smooth vectors in $\mathfrak{g}$ -module V, i.e. those vectors killed by any product $(\epsilon c_1)\dots(\epsilon c_k)$ of a fixed number k of elements of the form ϵc in $A((\epsilon)) \otimes \mathfrak{g} \subset U(\tilde{\mathfrak{g}}_A)$ (p.909)

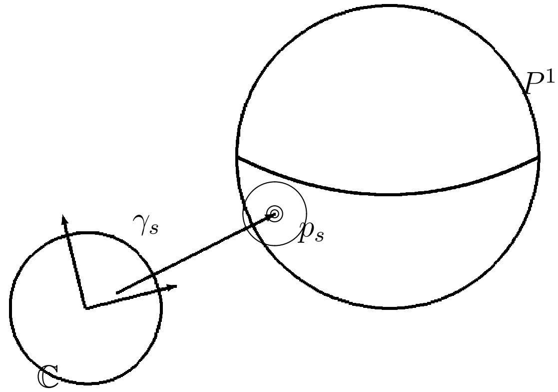
$\mathbf{V}^\kappa_a$ (2.4) Weyl module induced by the irreducible module $\mathcal{V}_a$

$[V]$ (p.396) $\sum_{a\in F^t} \chi^t(V, a) \cdot [\mathbf{V}^\kappa_a]$

$\underline{\mathscr{V}} = H\backslash\mathscr{V}$ (sec.9.2) moduli space of complex curves of genus zero with marked points and fixed local charts (which can be extended almost everywhere) at the marked points

$\underline{V}_s$ (8.3) where $s \in \spadesuit$, is a collection of $\tilde{\mathfrak{g}}_B$ modules with the same central charge $\kappa - h$

$\mathscr{V}$ (9.1, p.950) the open subset of $H^S = \overbrace{H \times \cdots \times H}^{S}$, to be pictured as the configuration space of charts $\mathbb{C} \to P^1$ with distinct targets for their origins $0 \in \mathbb{C}$, or, configuration space of a collection S of points in P^1, each point being accompanied by a local chart



$\mathscr{V}'$ (9.1) a configuration space similar to $\mathscr{V}$, but with an extra point added, without an accompanying chart, i.e. $\mathscr{V}'$ is an open subset in $H^S \times P^1$.

$\underline{\mathscr{V}}$, $\underline{\mathscr{V}}'$ (9.2) quotients of, respectively, $\mathscr{V}$ and $\mathscr{V}'$ by the left action of $H = PGL_2(\mathbb{C})$

$\underline{V}_s$ (9.11, p.953) a smooth $\tilde{\mathfrak{g}}_B$-module with central charge $\kappa - h$

$\underline{\mathscr{V}}_0$ (13.1) is a contractible real analytic subset of $\underline{\mathscr{V}}$

v (19.1) $= e^{i\pi\varpi} \in A$

v_i (19.1) $= v^{\delta_i}$

V_a (19.3, p.337) finite dimensional irreducible module of highest weight a

${}^0V$ (19.8) a vector subspace in V such that $V = {}^0V \otimes_{\mathbb{C}} \mathbf{A}$.

V_+ (Section 26, p.384) is defined as $Q_1^{\#}V$

V_0 (section 26) is the $\tilde{\mathfrak{g}}^+$-invariant subspace of V such that $V = V_+ + V_0$

$\beta(V_1, V_2)$ (remark after 26.4) is the natural transformation (isomorphism) $\mathscr{G}(V_1) \otimes \mathscr{G}(V_2) \to \mathscr{G}(V_1 \dot{\otimes} V_2)$

V_i (26.4) is defined as Weyl modules $\mathbf{V}^{\kappa}_{a_i}$

$\mathbf{V}_{a,m}$ (393) is defined as ${}_{\lambda_a(m)}\mathbf{V}_a^\kappa$

$\mathbf{V}_a^{\hat{\kappa}}$ (406) is defined as $\mathbf{V}_a \otimes_R R_\kappa$ (remark after corollary to lemma 29.12)

$\bar{V}_i$ (414) quotient of two successive objects V_i in a composition series

$\mathbf{V}^N$ is defined as $(V_c^\kappa)^{\otimes N}$

$\mathbf{V}_a$ (421)

$\check{V}$ (421)

$V \sim_a 0$ (424) a-equivalence; V is a-equivalent to zero if $Hom_{\mathscr{O}_{\kappa_0}}(V^{\kappa_0}_{\bar{a}-\bar{\alpha}_i}, V) = \langle 0 \rangle$.

$V^{(a,b)}$ (425) is defined as $Hom_{\check{\mathscr{O}}}(V_{\bar{a}}\dot{\otimes}V_b, V)$

$\check{v}_i$ (427) is an automorphism of the functor $\mathbf{X}$

$\mathscr{V}_c^\lambda$ (429) is defined as $\{v \in \mathscr{V}_c | h_i v = \lambda(i)v; i \in I\}$

W (4.4, p.926) $= \bigotimes_{s\in\spadesuit} V_s$, where V_s are $\tilde{\mathfrak{g}}$ modues, with a natural filtration coming from the filtration of $U(\Gamma)$

$\hat{W}$ (4.8, p.927) the projective limit of the system of vector spaces $W/G_1W \leftarrow W/G_2W \leftarrow W/G_3W \leftarrow \dots$ induced by the decreasing filtration $W \supset G_1W \supset G_2W \supset \dots$

W_1 (7.4) $= \bigotimes_{s\in\spadesuit} V_s(N_s)$ where the $N_s \geq 1$ (7.1, p.938)are integers such that $V_s(N_s)$ are finite-dimensional and generate V_s over $\tilde{\mathfrak{g}}$ (Theorem 2.22, p.918); This vector space is introduced in the demonstration of the finite-dimensionality of the quotient W/G_MW.

$\underline{W}$ (8.3, p.943) is the tensor product $\bigotimes_{s\in\spadesuit} \underline{V}_s$ over B; It has a structure of a $\hat{\mathfrak{g}}_B$ module and hence a structure of a Γ_B module via the homomorphism $\Gamma_B \to \hat{\mathfrak{g}}_B^\heartsuit$ (8.2(b); c.f. 4.6(a), p.926)

$\underline{W}$ (8.3) is the projective limit $\varprojlim_N W/G_NW$

$W_n, \hat{W}_n, \hat{W}$ (15.19) for $W = \bigotimes_{i=1}^4 V_i$, we have: $W_n = B_n \otimes W$; and $\hat{W}_n = \varprojlim_N W_N/G_NW_n$; and $\hat{W} = \varprojlim_N W/G_NW$

W (19.1) the Weyl group, i.e. the finite subgroup of $Aut(\mathbb{Z}^I)$ generated by the reflections $s_i : s_i(\omega_j) = \omega_j - \delta_{ij}\alpha_i$ for $j \in I$.

w_0 (19.1) longest element of the Weyl group; $w_0(\omega_i) = -\omega_{\bar{i}}$ for every $i \in I$.

W (393) the Weyl group for $\mathfrak{g}$

W_κ (393) a certain subgroup of the group of affine automorphisms of $\mathbb{Z}^I$, depending on the rationality of κ

$\tilde{W}$ (412) is defined as the semidirect product $W \rtimes P$, the affine Weyl group

W_κ (412) a subgroup of the affine Weyl group, depending on the rationality of κ

W_a^κ (412) the unique direct summand of $V_a^\kappa \dot{\otimes} V_{\bar{a}}^\kappa$ which has a quotient isomorphic to $\mathbf{1}$. It is pertinent to the numerical criterion of rigidity, proposition 31.1.

$W^{(a,b)}$ (428) is defined as the complex vector space $Hom_{\mathscr{O}_\kappa}(V_{\bar{a}}^\kappa \dot{\otimes} V_b^\kappa, W)$.

$\tilde{\mathbf{X}}$ (p.438, theorem 38.1) a braided functor giving the equivalence of categorories $\mathcal{O}_\kappa \to \mathscr{C}_\kappa$

X_N (7.3) the complex subspace of $U(\Gamma)$ spanned by the products $(g(s_1)c_1)(g(s_2)c_2)\ldots(g(s_N)c_N)$ with $c_i \in \mathfrak{g}$ and $s_i \in \heartsuit$. We have $X_N \subset G_N$.

X (Theorem 7.9) is a smooth $\tilde{\mathfrak{g}}^\heartsuit$ module where $\mathbf{1}$ acts as the scalar $\kappa - h$

$\underline{X}$ (8.5) is a smooth $\tilde{\mathfrak{g}}_B^\heartsuit$ module where $\mathbf{1}$ acts as the scalar $\kappa - h$

ξ (10.13) is an element $\xi = \sum_{j,s} a_{j,s}\theta_{j,s} \in \mathscr{D}_2$; $a_{j,s} \in A$. The coefficients $a_{0,s}, a_{-1,s}$ satisfy a relation with the coefficients $l_{ss'}$.

X_{12}, X_{34} are tensor products $V_1 \dot{\otimes} V_2$ and $V_3 \dot{\otimes} V_4$ respectively

X_n (16.10, p.989) Beilinson's "diagonal module" $X_n = U(\tilde{\mathfrak{g}}_{B_n}^\heartsuit) \otimes_{U(P)} B_n$

x_a (19.4) $= \theta(y_a)$. It satisfies relations $f_i(x_a) = 0$ and $h_i(x_a) = -\bar{a}(i)x_a$.

$\mathscr{X}$ (19.8, p.338) space of analytic (in the sense of §19.8, p.338) solutions on the interval $(0,1)$ with values in vector space V of the (formal KZ) equation $\frac{df}{dz} = \frac{\varpi\Pi_0 f}{z} + \frac{\varpi\Pi_1 f}{z-1}$ (19.8(a)).

$\mathbf{X}_\lambda(V)$ (426) is defined as $\varinjlim_{b-a=\lambda} V^{(a,b)}$

$\mathbf{X}(V)$ (426) is defined as $\bigoplus_{\lambda\in\mathbb{Z}^I} \mathbf{X}_\lambda(V)$

$\bar{\mathbf{X}}_\lambda(W)$ (428) is defined as $\varinjlim_{b-a=\lambda} W^{(a,b)}$

$\bar{\mathbf{X}}(W)$ (428) is defined as $\bigoplus_{\lambda\in\mathbb{Z}^I} \bar{\mathbf{X}}_\lambda(W)$

x_c^0 (429) is an element in $(V_{\bar{c}}^\kappa)^{(c,0)}$

$\mathbf{X}(V)(\kappa)$ (432) is defined as $\mathbf{X}(V)/m_\kappa\mathbf{X}(V)$

$\tilde{\mathbf{X}}$ (437) is the functor

$$W \longrightarrow \tilde{\mathbf{X}} = \left(\bar{\mathbf{X}}(W) = \bigoplus_\lambda \bar{\mathbf{X}}_\lambda(W), (\bar{E}_i^W)^{(n)}, (\bar{F}_i^W)^{(n)}\right)$$

(see corollary to prop. 36.1). The pair $\tilde{\mathbf{X}} = (\bar{\mathbf{X}}, \bar{M})$ is a braided functor: $\mathscr{O}_\kappa \to \mathscr{C}_\kappa$ (lemma 38.1, p.437). It turns out to be an equivalence of categories (Theorem 38.1, p.438).

$\mathbf{X}_c$ (431) is defined as $\bar{\mathbf{X}}_c(V_c^\kappa)$

$\mathbf{X}_c^*$ (431) is defined as the object $\bar{\mathbf{X}}_c((V_c^\kappa)^*)$ in $\bar{\mathscr{E}}_\kappa$

Y (9.11, p.953) $= \bigotimes_{s\in S} \underline{V}_s$

Y_{Δ_B} (9.11) the space of coinvariants $Y/\Delta_B Y$

Y_n (15.10, p.979) $= A_n \otimes \left(\bigotimes_{i=1}^4 V_i\right)$, a $\hat{\mathfrak{g}}_{A_n}^S$-module and hence a $\Delta_{4,n}$-module

y_a (19.3) highest weight vector of V_a; we have $e_i y_a = 0, h_i y_a = a(i) y_a$.

$\bar{Y}$ (26.4) is defined as $\bigotimes_i \mathscr{V}_{a_i}$

Y (26.4) is defined as $\bigotimes_i \mathbf{V}_{a_i}^\kappa$

y_c^0 (429) is an element in $(V_{\bar{c}}^\kappa)^{(0,c)}$

$Y/\Delta Y$ (969) where $\Delta = A' \otimes \mathfrak{g}'$ (p.963), is an A-module, and it is projective and of finite rank (p.969)

Z (section 6) is the vector space $Z = Hom_{\mathbb{C}}(W, \mathbb{C})$

Z (6.2) is the space of linear forms $Hom_{\mathbb{C}}(W, \mathbb{C})$, with the action of Γ from W

Z^N (6.2, p.934) annihilator of $G_N W$, i.e. $Z^N = \{\lambda \in Z | (\forall \xi \in G_N) \xi\lambda = 0\}$

Z^∞ (6.2) the union of all Z^N, i.e. the inductive limit of $Z^1 \subset Z^2 \subset Z^3 \subset \dots$

Z (393) set of W_κ orbits on the weight lattice $\mathbb{Z}^I$.

Z_r (421) a right-infinite subset of the real line

ζ (436) is defined as $exp(-i\pi/\delta\kappa)$

$\dot{\otimes}$ (section 13.4, p.972, 973, etc) the fusion tensor product (sections 4-7, section 13.4, see also part IV), which is the tensor product structure in the category $\mathcal{O}_\kappa$ of modules over an affine Lie algebra $\mathfrak{g}$ with level $\kappa - \check{h}$

$^\#$ (1.5) the involution of the affine Lie algebra $\tilde{\mathfrak{g}}_A$ given by $(\epsilon^n c)^\# := (-\epsilon)^{-n} c$ and $(\mathbf{1})^\# := -\mathbf{1}$

$[V]$ (p.396) $\sum_{a \in F^t} \chi^t(V, a) \cdot [\mathbf{V}_a^\kappa]$

$\langle V_1, V_2 \rangle$ (2.32, p.921) $= (V_1 \otimes V_2)/\tilde{\mathfrak{g}}(V_1 \otimes V_2)$, the $\mathbb{C}$-vector space of coinvariants of $\tilde{\mathfrak{g}}$ modules V_1 and V_2. There is an isomorphism (2.32(c))

$$Hom_{\mathcal{O}_\kappa}(V_1, D(V_2)) \cong Hom_{\mathbb{C}}(\langle V_1, V_2 \rangle, \mathbb{C}).$$

$\langle V_1, V_2, \dots, V_r \rangle = \mathcal{S}(E)$ (13.3, p.971) space of horizontal analytic sections in the A-module bundle $Y/\Delta Y$ of coinvariants over the moduli space $\underline{\mathscr{V}}$ with a natural connection given by Sugawara operators (sections 11,12)

$\clubsuit, \spadesuit, \heartsuit$ (4.2,p.925) are sets of points on the genus zero complex curve under discussion; in particular, there is just one point $s \in \heartsuit$ on each component of the curve, and there are at least two points (denoted $[s]$) from $\spadesuit$ on each component; these $\{[s] \mid s \in \heartsuit\}$ partition the set $\spadesuit$

$[V : \mathbf{L}^{\kappa}_{a'}]$ (3.9, p.924) the number of subquotients isomorphic to $\mathbf{L}^{\kappa}_{a'}$ in the composition series of V

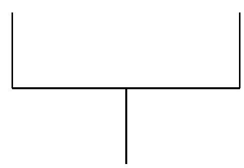
[12] is the set {1,2,12}.

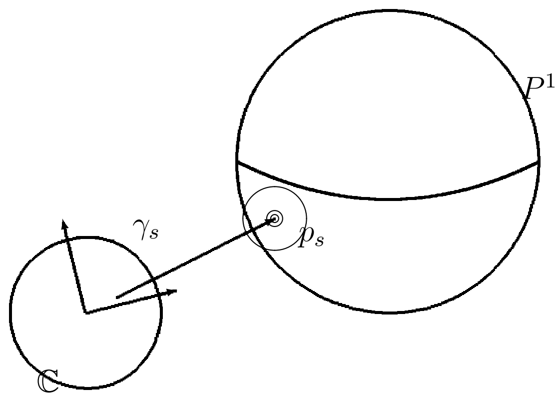
$\langle\rangle$ (19.1) pairing in the weight lattice: $\langle a, b\rangle = \sum_{i,j} \delta_i b_{ij} a(i) b(j)$

$[n]_i$ (19.1) $= \frac{v_i^n - v_i^{-n}}{v_i - v_i^{-1}}$

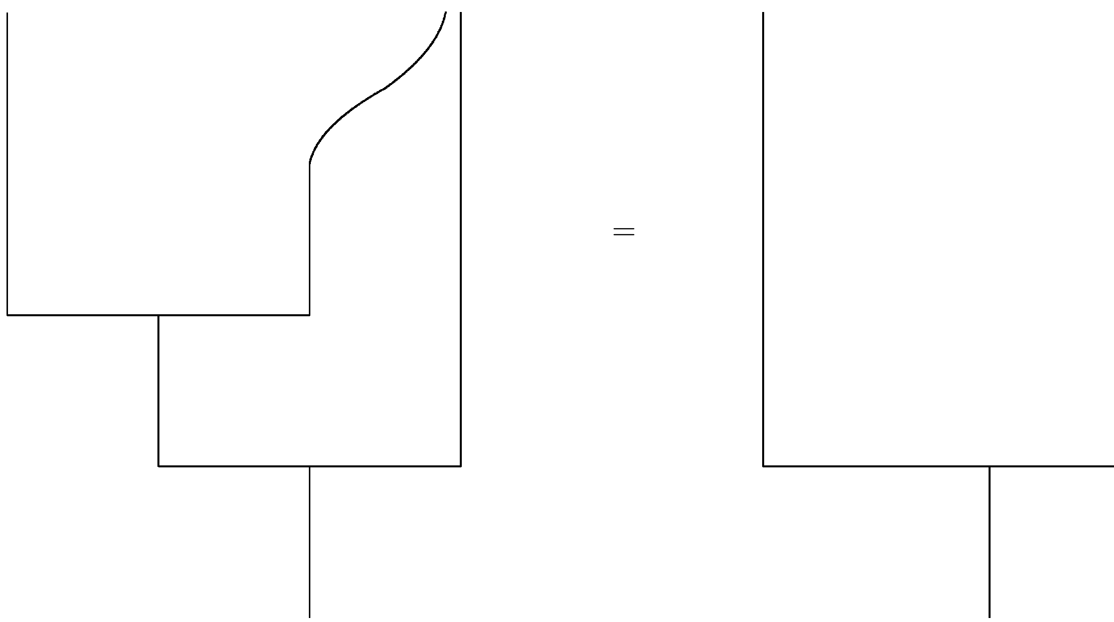
$[n]$ (433) is defined as $\frac{v^n - v^{-n}}{v - v^{-1}}$ for an integer n

$(,)$ (435) is a bilinear form on $\mathbf{f}$





P^1
γ_s
p_s
$\mathbb{C}$



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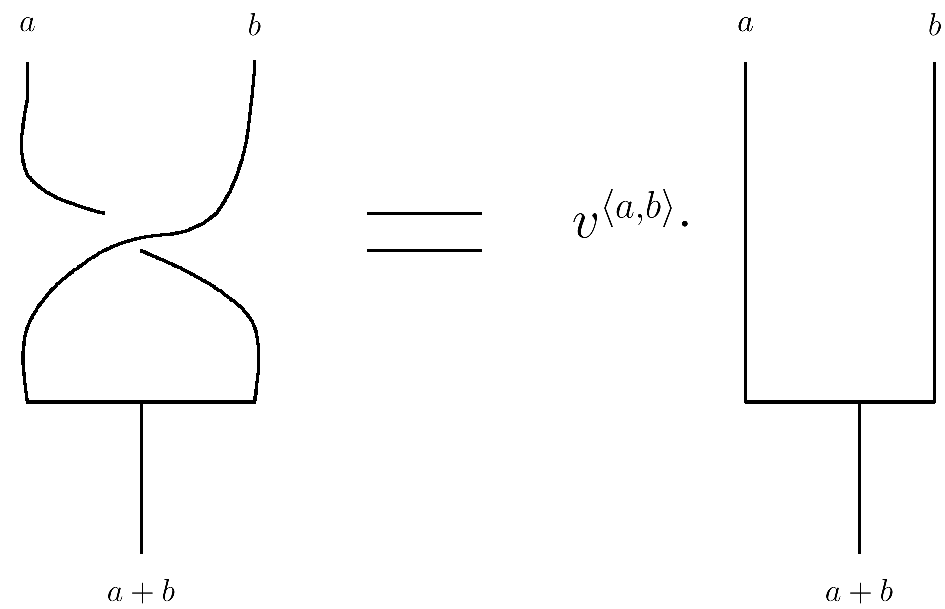
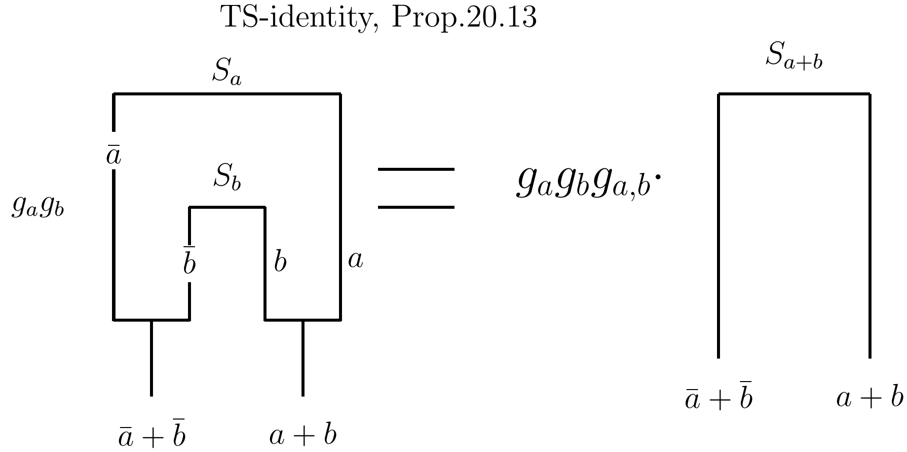
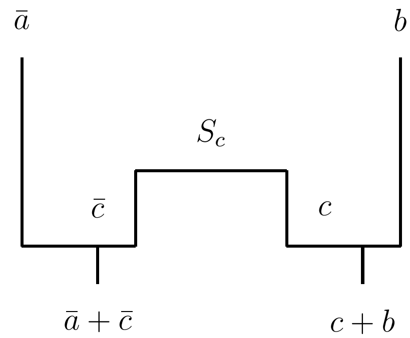
TC-identity, Prop.20.3



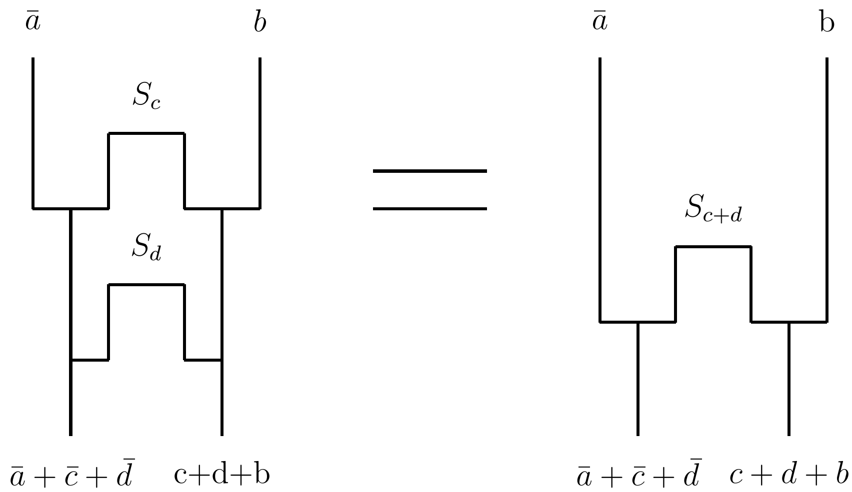
TS-identity, Prop.20.13



Morphism $tr^c_{a,b}$, §21.1

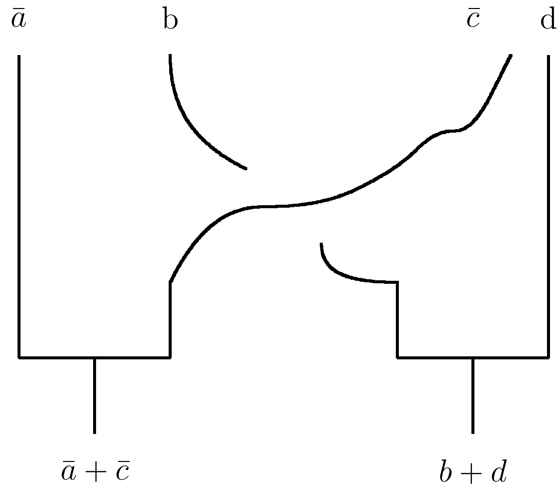


Transitivity of tr, Prop.21.3

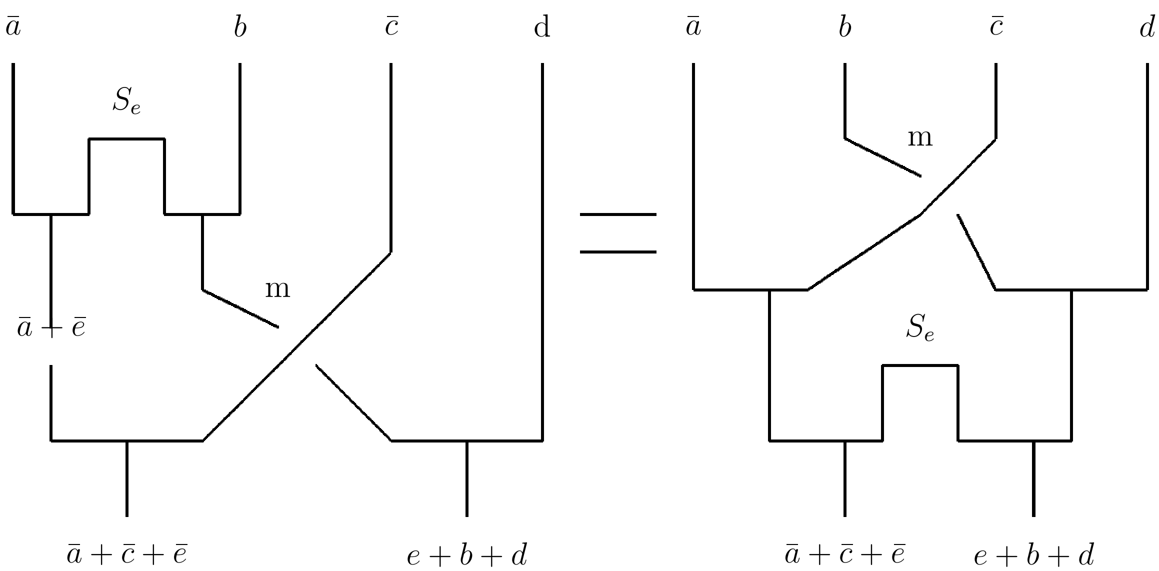


m-morphism (§21.4)

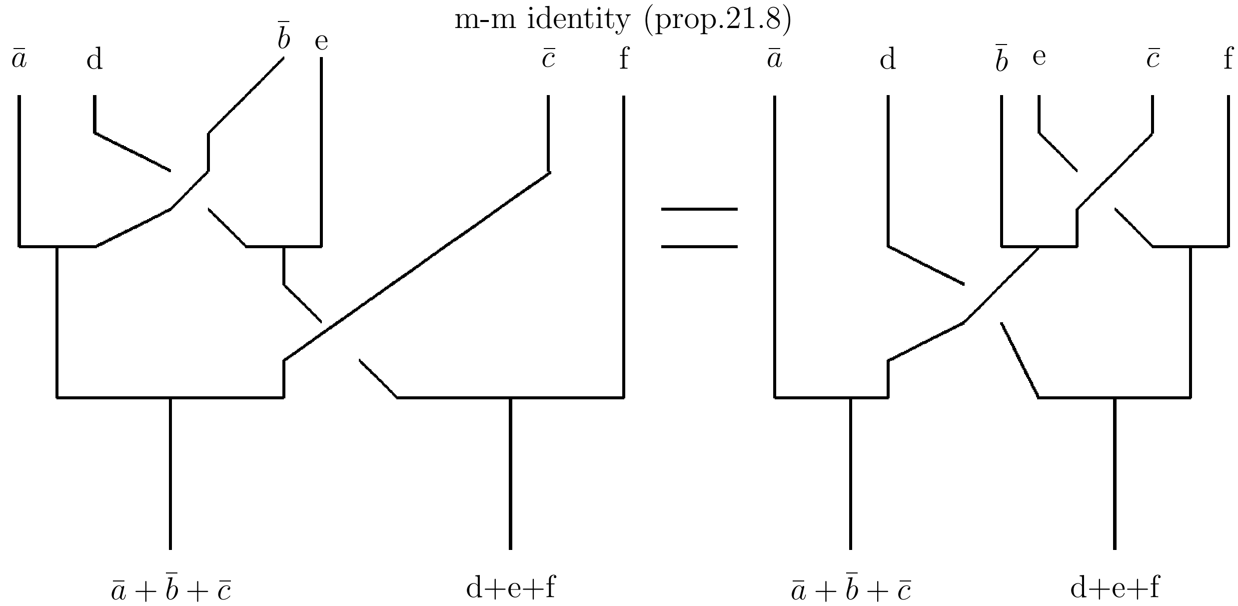
$v^{\langle b,c\rangle}$:

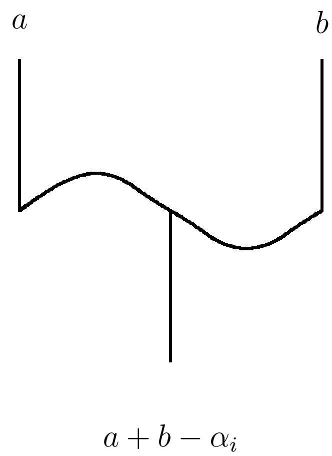


Left compatibility of m- and tr-morphisms, Prop.21.6



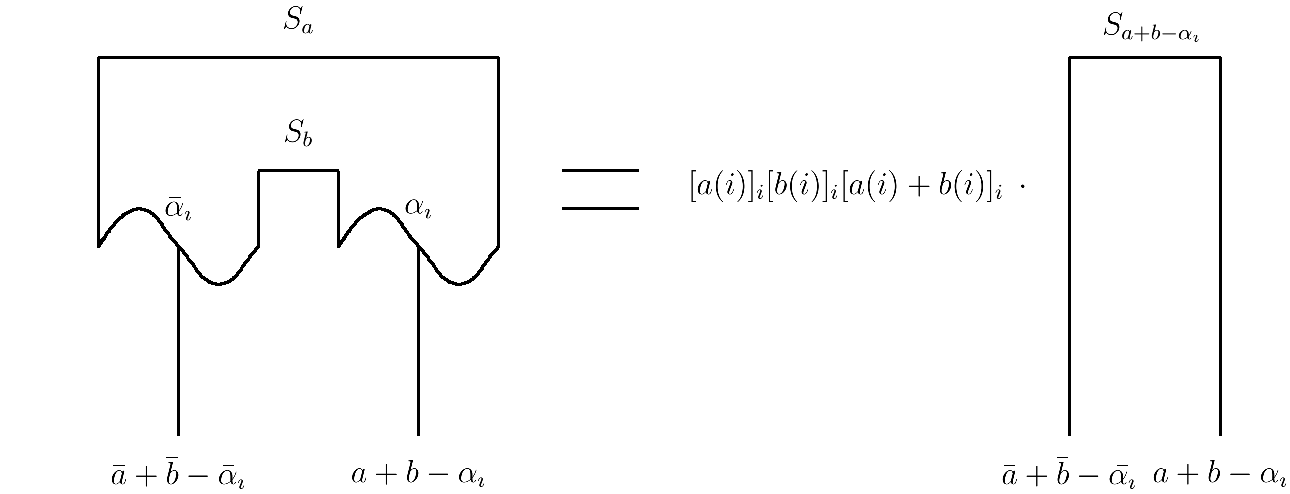
m-m identity (prop.21.8)



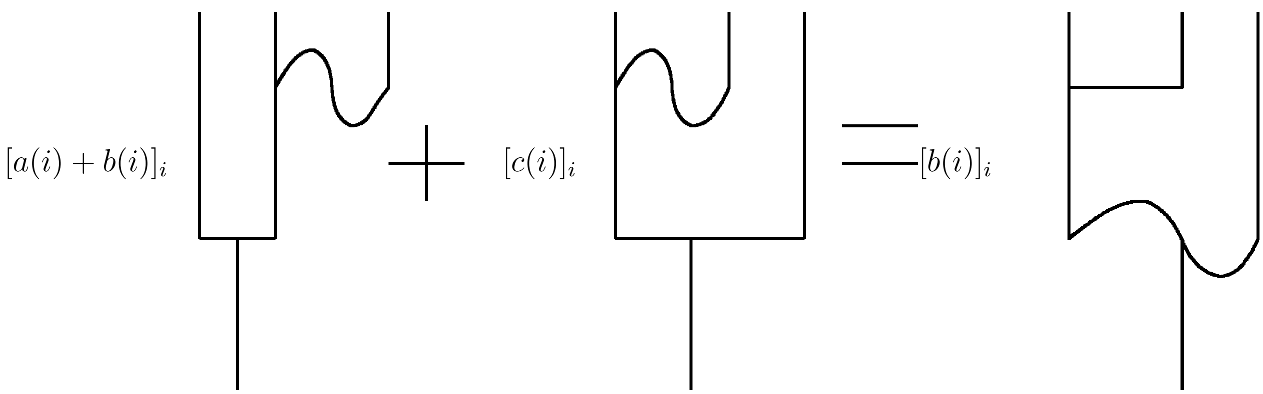


$\tau'_{i;a,b}: V_{a+b-\alpha_i} \to V_a \otimes V_b$ (22.1)

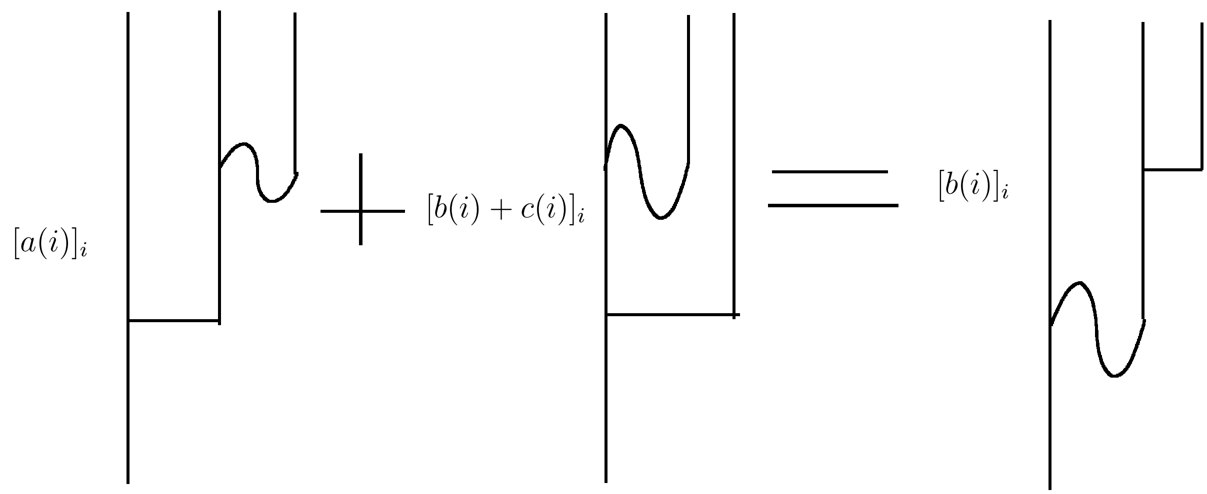
$\tau - S$-identity, §23.7



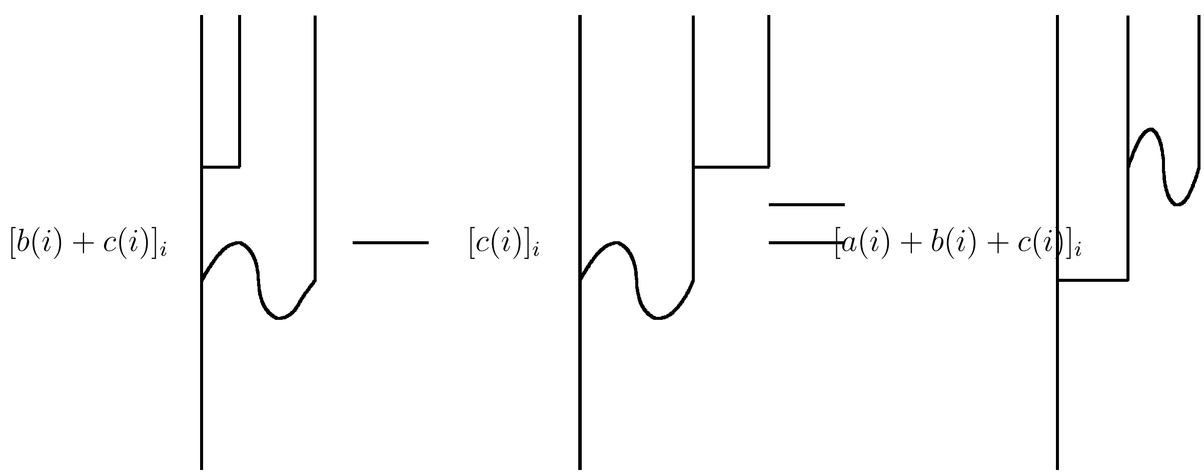
τT-identity (1), Prop.22.6



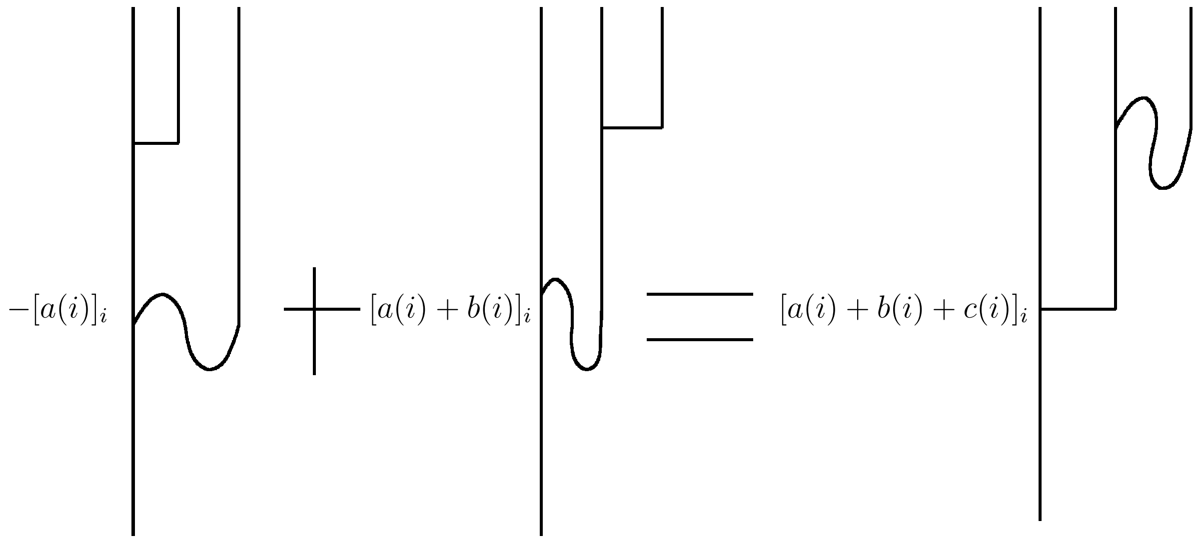
$\tau - T$-identity (2), Prop.22.6



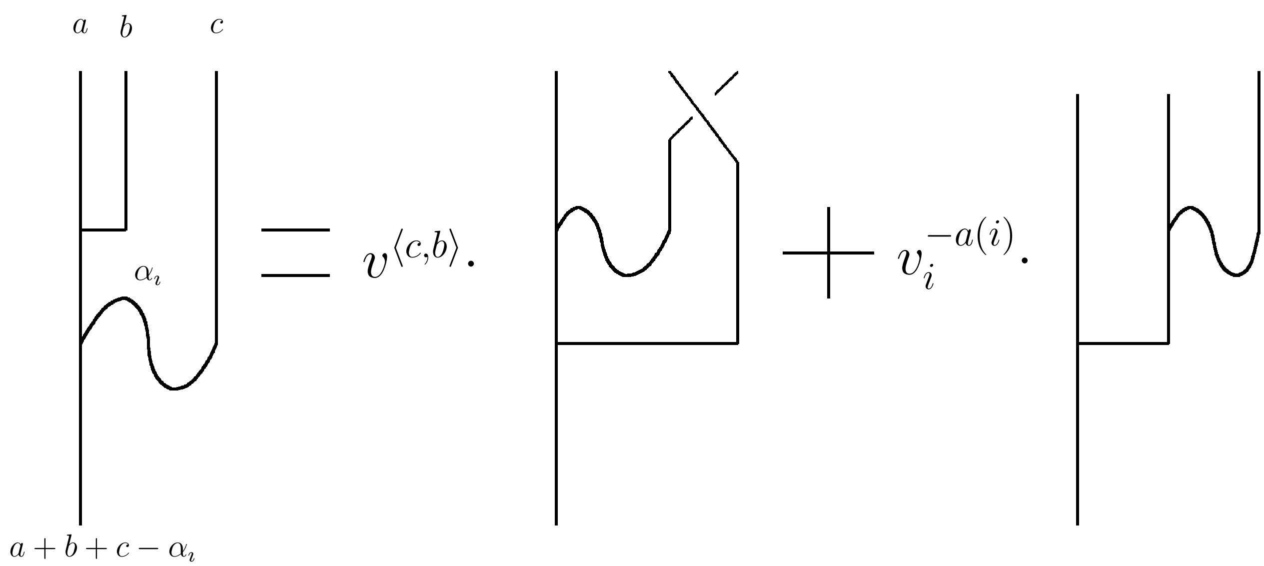
$\tau - T$-identity (3), Prop.22.6



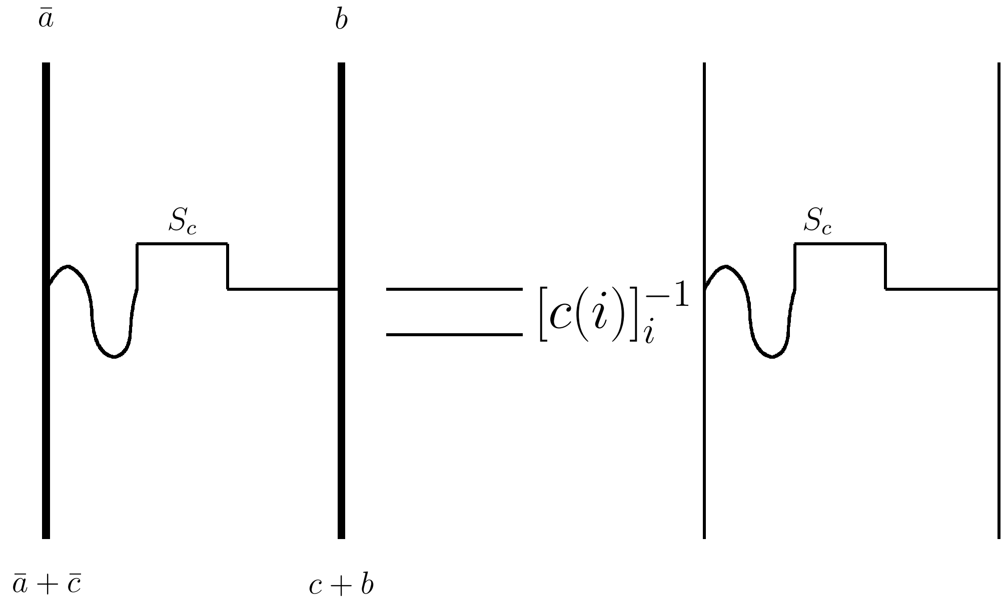
$\tau - T$-identity (4), Prop. 22.6



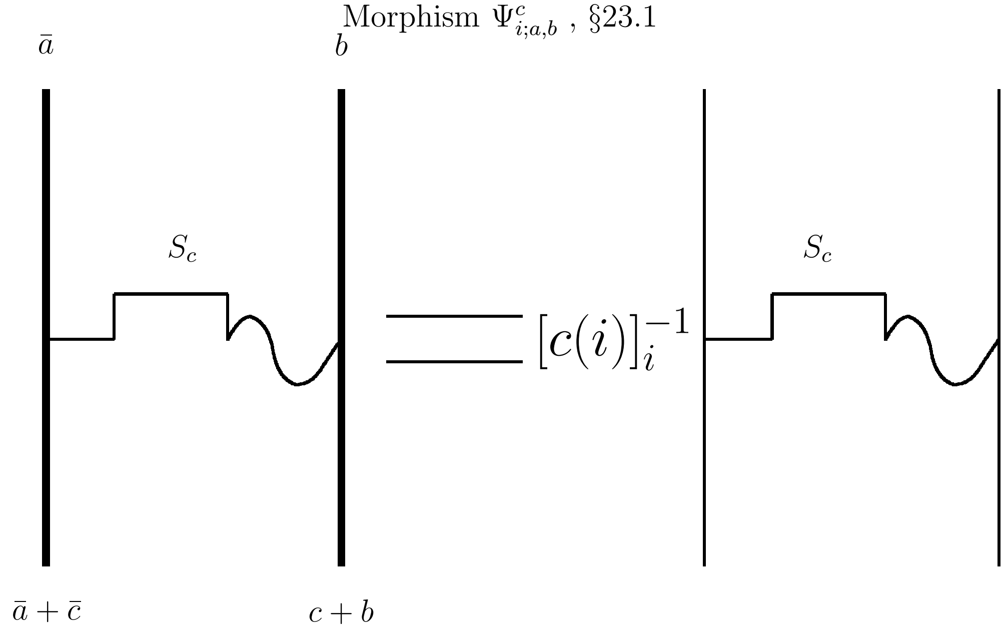
A relation for τ, T and C , Cor. 22.9



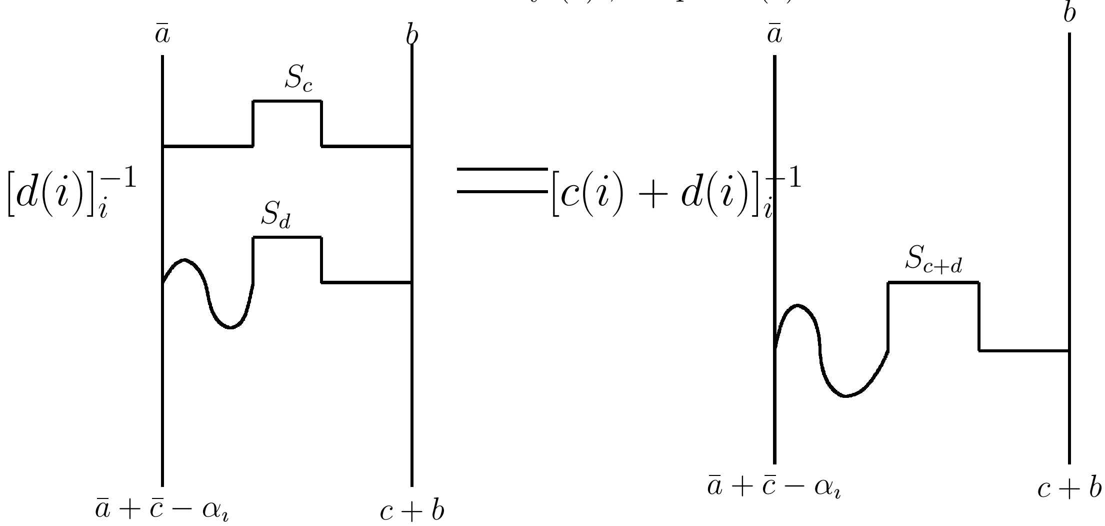
Morphism $\Phi^c_{i;a,b}$, §23.1



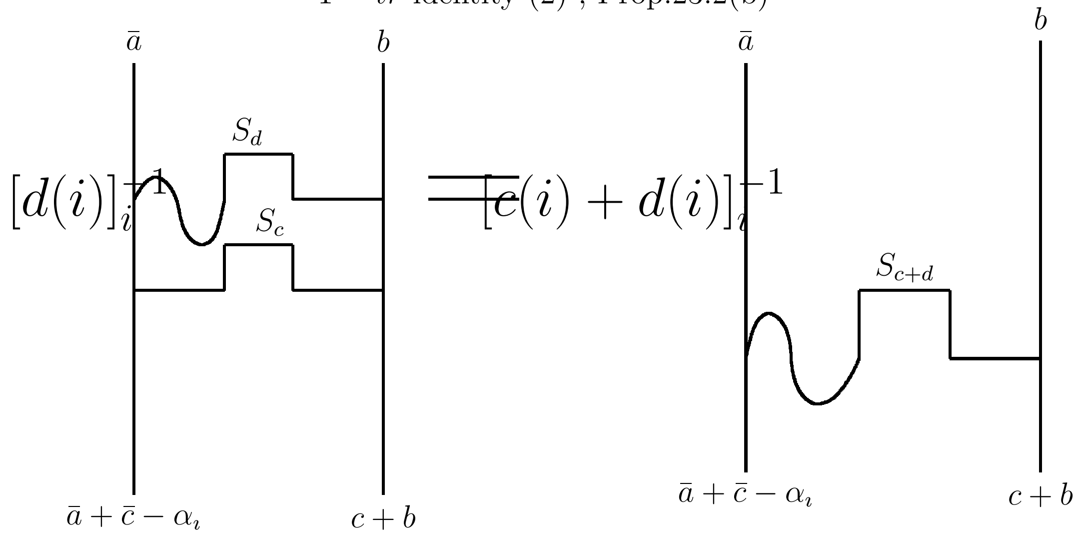
Morphism $\Psi^c_{i;a,b}$, §23.1



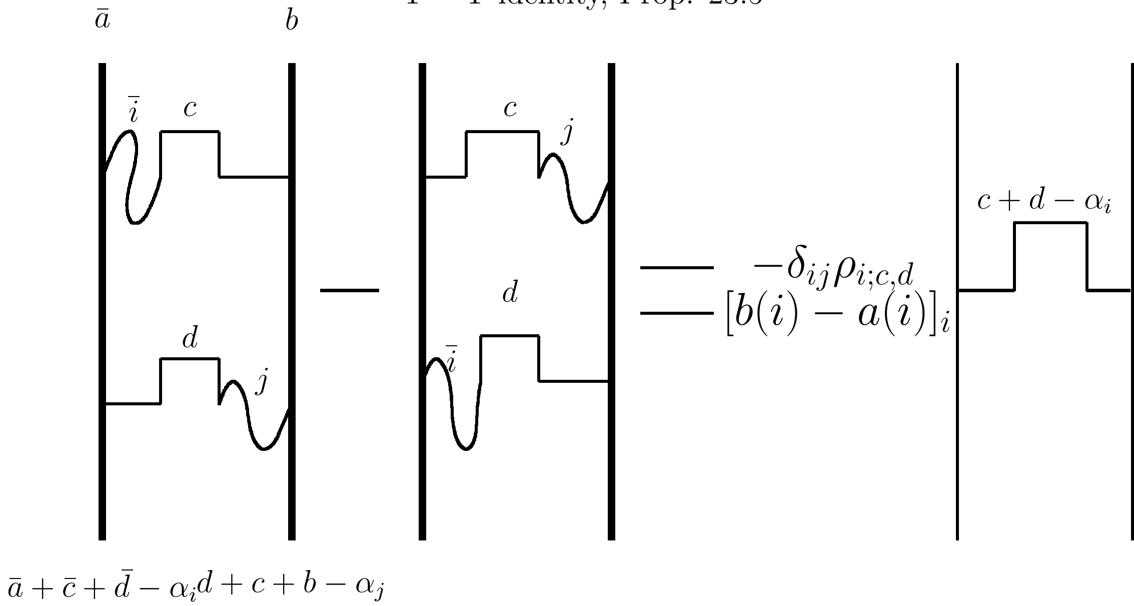
$\Phi - tr$-identity (1) , Prop.23.2(a)



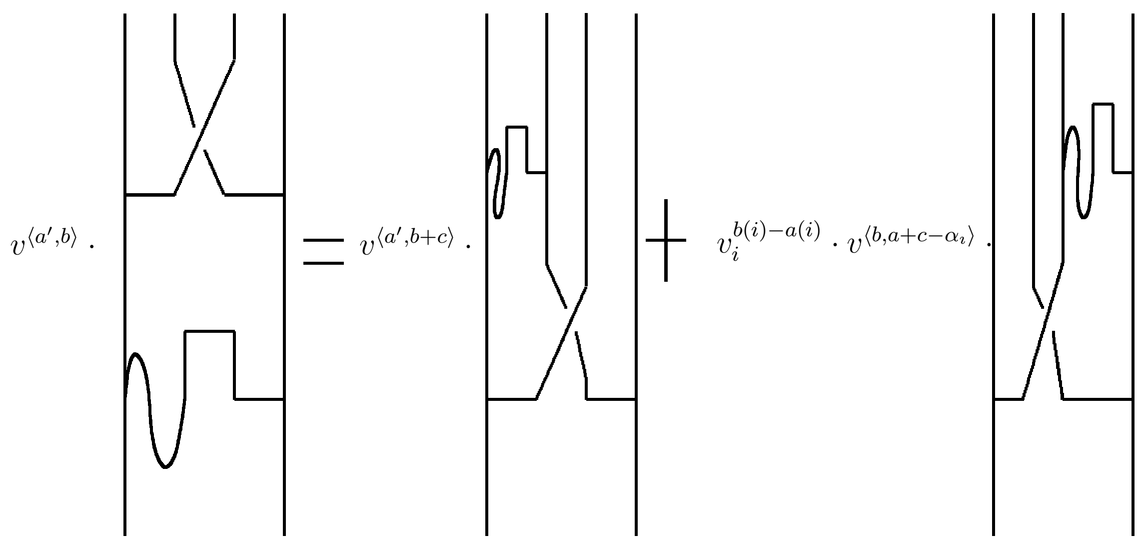
$\Phi - tr$-identity (2) , Prop.23.2(b)



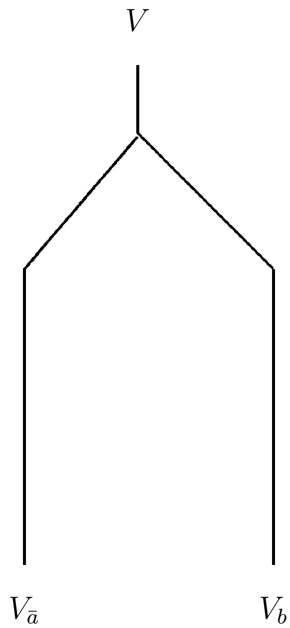
$\Phi - \Psi$-identity, Prop. 23.5



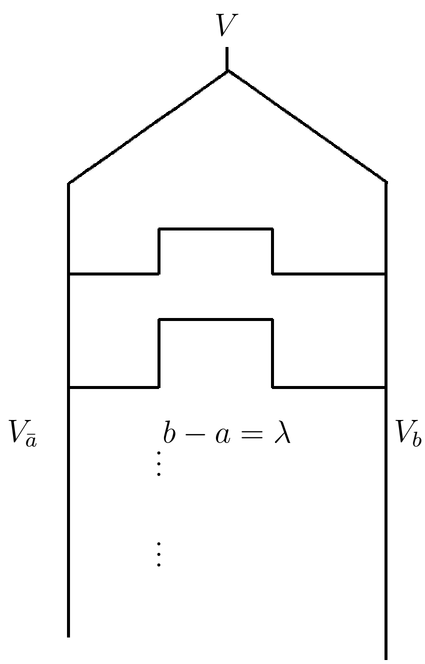
$\Phi - m$-identity, Prop. 23.9



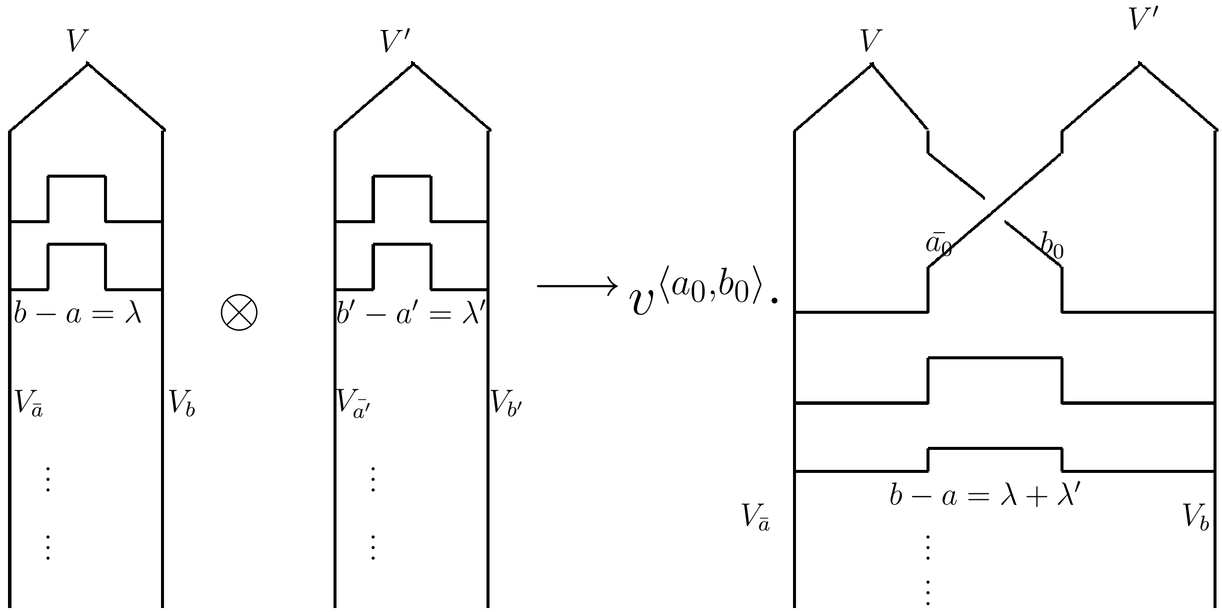
$V^{(a,b)} := Hom_{\mathscr{D}}(V_{\bar{a}} \otimes V_b, V)$, §25.1



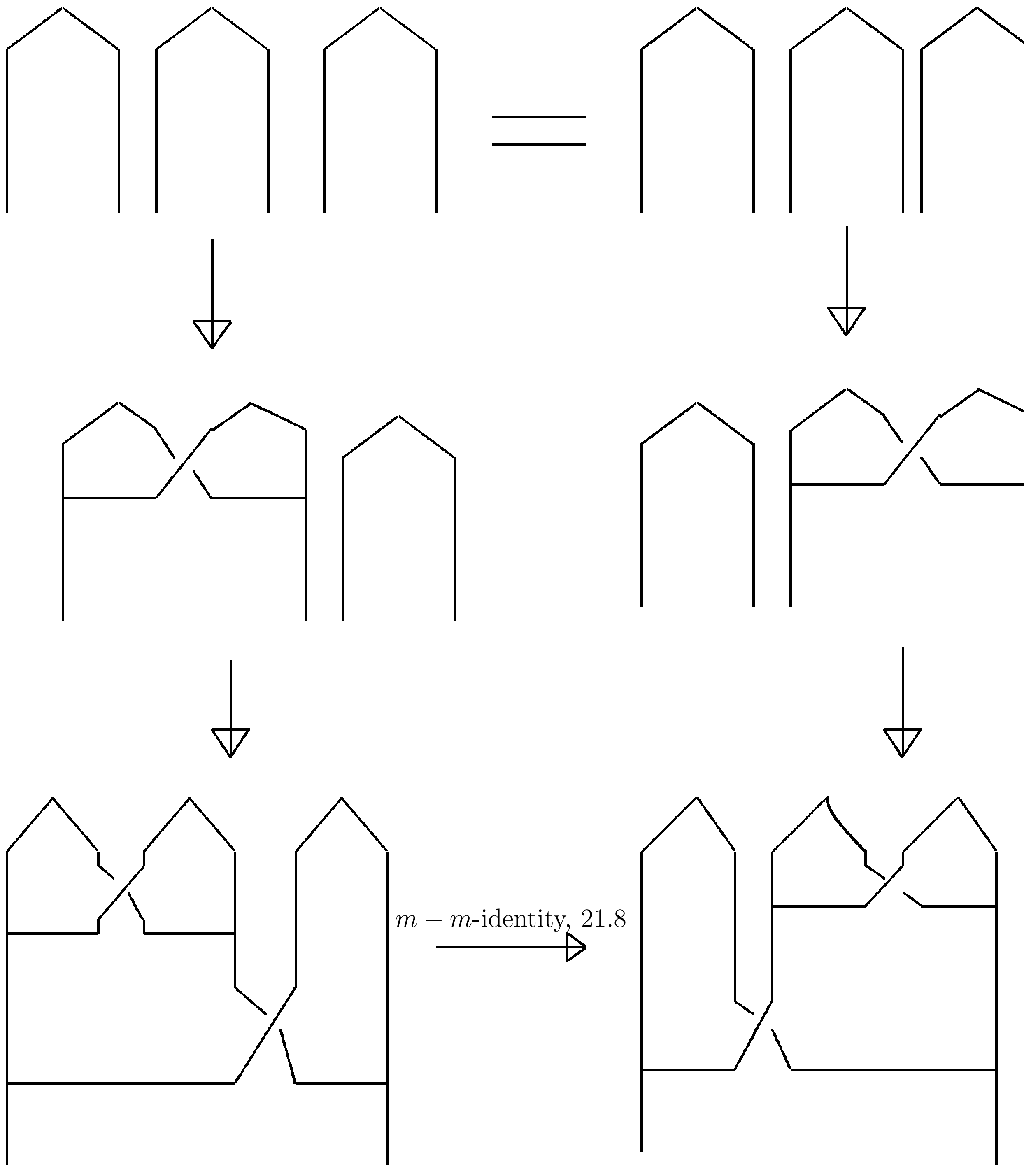
$X_\lambda(V)$



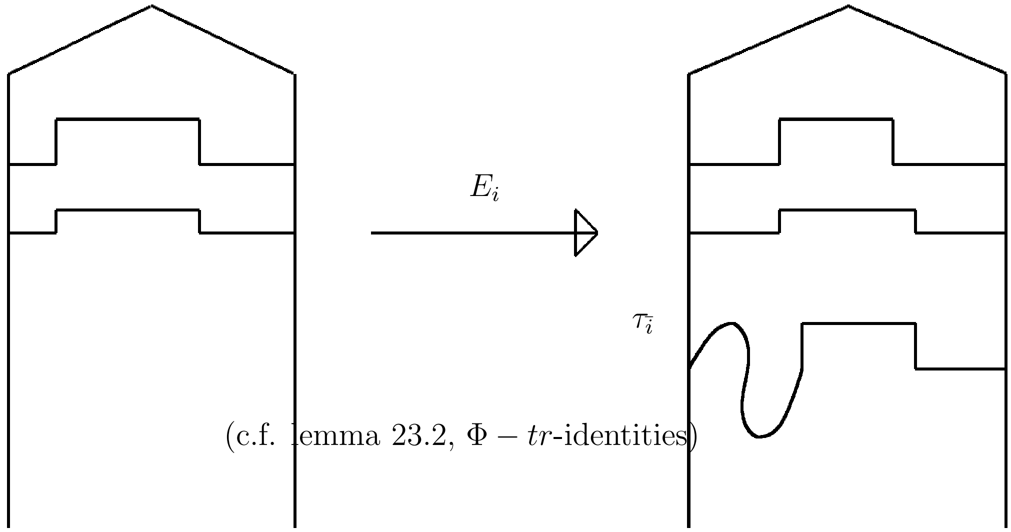
The tensor product $X_\lambda(V) \otimes X_{\lambda'}(V') \to X_{\lambda+\lambda'}(V \otimes V')$, 25.5(a)



Associativity of the functor X, Prop.25.8



The morphism E_i, representing a quantum group element



(c.f. lemma 23.2, $\Phi - tr$-identities)