1 List of the many objects (symbols) in the series of papers on Tensor structures by Kazhdan and Lusztig

Here we list the objects (or symbols) used by Kazhdan and Lusztig in their series of paper "Tensor Structures Arising from Affine Lie Algebras, I-IV", published in the Journal of the American Mathematical Society, 1993-1994.

$r_{a,b}$ (10.16(a)): coefficients for the change of basis from $\Theta_{a-b,s}$ to σ_k induced by the isomorphism of the space $H^S \simeq H^S$ given by (γ_s)

$T_{a,b}$ (20.1): a morphism $V_{a+b} \to V_a \otimes V_b$ of modules over $\mathfrak{g}$

S'_c (20.4): a morphism: $V_{\bar{c}} \otimes V_c \to V_0 \simeq A =$ the ring of formal power series over a variable (which will turn out to be $1/(k+\check{h})$, the level plus the dual coxeter number), whose value at the highest weight vector $x_{\bar{c}} \otimes y_c$ is normalised as 1

S_c (20.12): the morphism S'_c normalised by an "admissible collection" (20.11) of coefficients $\{g_a\}$ satisfying some cocycle conditions with the set $\{g_{a,b}\}$ (20.6(a))

$g_{a,b}$ (20.6(a)): elements in $\mathbf{A}$, measuring the discrepancy of the S'_c morphism at different weight c's before and after the action of the morphism $T_{a,b}$

V_a (19.3) highest weight module of $\mathfrak{g}$

$\{\omega_i | \, i \in I\}$ (19.1) the collection of simple roots

s_i (19.1) simple reflections

$\bar{i}$ (19.1) is the label for the corresponding element in the root system for α_i in the involution induced by the longest element w_0 of the Weyl group of $\mathfrak{g}$.

t (19.2(c)): $t = (1/2)(\Delta(\Omega) - 1 \otimes \Omega - \Omega \otimes 1)$, a bilinear operator on $\mathfrak{g}$ -modules which commutes with the action of $\mathfrak{g}$

$\mathcal{O}_\kappa$ (Kazhdan and Lusztig's introduction, p.905) - "category O", consisting of modules of finite length over $\hat{\mathfrak{g}}$, with central charge $\kappa - h$, and whose composition factors are irreducible highest weight modules corresponding to weights which are dominant in the $\mathfrak{g}$ -direction (p.905)

Γ (p.926) is the Lie algebra $\mathfrak{g}$ with coefficients in the ring of regular functions over a genus-zero complex curve with finitely many points removed

$\mathscr{A}_k$ (section 27.1, p.390), the full subcategory in $\mathcal{O}$ of modules having a Weyl filtration or other projectivity or ext conditions

$\dot{\otimes}$ (section 13.4, p.972, 973, etc) the fusion tensor product (sections 4-7, section 13.4, see also part IV), which is the tensor product structure in the category $\mathcal{O}_\kappa$ of modules over an affine Lie algebra $\mathfrak{g}$ with level $\kappa - \check{h}$

$T(W)^{\#}$ (sections 4.4,4.11, p.926, p.930) a construction of the fusion tensor product as a $\tilde{\mathfrak{g}}^{\heartsuit}$ module, from a tensor product $W = \bigotimes_{s \in \spadesuit} V_S$ where V_s is a $\mathfrak{g}$ module and $S = \heartsuit \sqcup \spadesuit$

$^{\#}$ (1.5) the involution of the affine Lie algebra $\tilde{\mathfrak{g}}_A$ given by $(\epsilon^n c)^{\#} := (-\epsilon)^{-n} c$ and $(\mathbf{1})^{\#} := -\mathbf{1}$

$V(\infty)$ (section 1.9) the collection of smooth vectors in $\mathfrak{g}$ -module V, i.e. those vectors killed by any product $(\epsilon c_1) \ldots (\epsilon c_k)$ of a fixed number k of elements of the form ϵc in $A((\epsilon)) \otimes \mathfrak{g} \subset U(\tilde{\mathfrak{g}}_A)$ (p.909)

Q_k (1.7) is the A-submodule of $U(\tilde{\mathfrak{g}}_A)$ generated by products $(\epsilon c_1) \ldots (\epsilon c_k)$, where $c_k \in \mathfrak{g}$

A (1.1) a commutative algebra with 1

$\mathcal{N}$ (2.1) nil-module, that is, a finite-dimensional module over the $\mathbb{C}$-algebra $\mathbb{C}[\epsilon] \otimes \mathfrak{g}$, and the ideal $\epsilon\mathbb{C}[\epsilon] \otimes \mathfrak{g}$ acts nilpotently on $\mathcal{N}$, i.e., there exists a fixed number $t > 1$ such that any t elements in $\epsilon\mathbb{C}[\epsilon] \otimes \mathfrak{g}$ acts on $\mathcal{N}$ as zero

$\mathcal{N}^\kappa$ (2.3) generalized Weyl module induced by the nil-module $\mathcal{N}$

$\mathbf{V}_a^\kappa$ (2.4) Weyl module induced by the irreducible module $\mathcal{V}_a$

Z (section 6) is the vector space $Z = Hom_{\mathbb{C}}(W, \mathbb{C})$

$M(\kappa)$ (section 29, p.397) is the quotient $M/(x - \kappa)$

$[V]$ (p.396) $\sum_{a\in F^t}\chi^t(V,a)\cdot[\mathbf{V}_a^\kappa]$

$\chi^t(V,a)$ (p.396) is the alternating sum $\sum_{i\in\mathbb{N}}(-1)^i\cdot dimExt^i_{\mathcal{O}^t_\kappa}(V,D(\mathbf{V}_a^\kappa))$

admissible filtration (pp. 391, 404) is a property satisfied by objects in $\mathcal{A}_\kappa$

tilting module (p.390) is a module in $\mathcal{O}_\kappa$ which is in $\mathcal{A}_\kappa$ (i.e. has a Weyl filtration) and so does its dual $D(M)$

$\tilde{\mathbf{X}}$ (p.438, theorem 38.1) a braided functor giving the equivalence of categorories $\mathcal{O}_\kappa\to\mathscr{C}_\kappa$

$\mathscr{V}=H\backslash\mathscr{V}$ (sec.9.2) moduli space of complex curves of genus zero with marked points and fixed local charts (which can be extended almost everywhere) at the marked points

f_s **and** f_s^k (sec. 9.5) $f_s:=\frac{1}{\gamma_s^{-1}(z)}\in\mathbb{C}$ is the reciprocal of the value of the local chart of a point z on the projective line $P(1)\cong\mathbb{C}$. The set $\{1,f_s^k(s\in S;k\geq 1)\}$ form a basis of A' over A (Lemma 9.6, p.951)

${}^sf$ (9.4) is a homomorphism of A-algebras: $A'\to A((\epsilon))$, which is an expansion along the extra parameter z of $\mathscr{V}'$ over $\mathcal{V}$ of a regular function $f\in A'=\mathbb{C}[\underline{\mathcal{V}'}]$ on the moduli space $\underline{\mathscr{V}'}=H\backslash\mathscr{V}'$ of the Riemann sphere with marked points

$\langle V_1,V_2\rangle$ (2.32, p.921) $=(V_1\otimes V_2)/\tilde{\mathfrak{g}}(V_1\otimes V_2)$, the $\mathbb{C}$-vector space of coinvariants of $\tilde{\mathfrak{g}}$ modules V_1 and V_2. There is an isomorphism (2.32(c))

$$Hom_{\mathcal{O}_\kappa}(V_1,D(V_2))\cong Hom_{\mathbb{C}}(\langle V_1,V_2\rangle,\mathbb{C}).$$

$\langle V_1,V_2,\ldots,V_r\rangle=\mathcal{S}(E)$ (13.3, p.971) space of horizontal analytic sections in the A-module bundle $Y/\Delta Y$ of coinvariants over the moduli space $\underline{\mathscr{V}}$ with a natural connection given by Sugawara operators (sections 11,12)

$\mathscr{D}_0$ (p.958) it is the complex vector subspace of $Der(\tilde{A})$ generated by $\theta_{0,s}$, $\theta_{+1,s}$, $\theta_{-1,s}$ for $s\in S$ (S=the configuration of points on the Riemann sphere)

$\mathscr{D}_1$ (p.958) it is a subspace of the set of derivations, which commute with the action of H, of the algebra $\tilde{A} = \mathbb{C}[\mathscr{V}]$ on the "lifted" configuration space $\mathscr{V}$ of marked points on the Riemann sphere, so that it maps surjectively $\mathcal{D}_1 \to Der(A)$ onto the space of derivations on the moduli space $\underline{\mathscr{V}}$

$\mathscr{D}_2$ (10.12, p.961) it is the Lie subalgebra of $\mathscr{D}_1$ consisting of derivations sending the subalgebra $A \subset \tilde{A}$ to zero, i.e. they are derivations along the fibres of the projection $\mathscr{V} \to \underline{\mathscr{V}}$; It is also an A-Lie algebra: the Lie algebra structure and the A-module structure on $\mathscr{D}_2$ are compatible. We have (12.11, p.969) $Der(A) = \mathscr{D}_1/\mathscr{D}_2 \to End_{\mathbb{C}}(Y/\Delta Y)$.

θ_k (10.2, p.957) vector fields on $H = PGL_2(\mathbb{C})$; where $k = 0, +1, -1$

$\theta_{k,s}$ (10.2) vector fields θ_k acting on the s-th tensor factor in a tensor product

$\clubsuit, \spadesuit, \heartsuit$ (4.2,p.925) are sets of points on the genus zero complex curve under discussion; in particular, there is just one point $s \in \heartsuit$ on each component of the curve, and there are at least two points (denoted $[s]$) from $\spadesuit$ on each component; these $\{[s] \mid s \in \heartsuit\}$ partition the set $\spadesuit$

$\dot{C}$, $\dot{C}'$ (17.1, p.992) are, respectively, the Riemann sphere with three points 0, 1, and ∞ removed, and the variety of (punctured) quadrics $\{(t, p, q) \in \mathbb{C}^3 | pq = t; p, q \neq 1\}$ over it

$\dot{A}$, $\dot{A}'$ (17.2, p.993) are spaces of regular functions on $\dot{C}$ and $\dot{C}'$ respectively

D, D' (15.2, p.977) are, respectively, the Riemann sphere with two points 1 and ∞ removed, and the variety of (punctured) quadrics $\{(t, p, q) | pq = t; p, q \neq 1\}$ over it

A_∞, A'_∞ (15.4) are space of regular functions over D and D' respectively

$\dot{\Delta}_4$ (17.5,p.994) the Lie algebra $\dot{A}' \otimes \mathfrak{g}$

$\mathscr{H}$ (17.8, p.995) a linear map on the tensor product $\bigotimes_i V_i$ defined by the Sugawara operators: $y_1 \otimes y_2 \otimes y_3 \otimes y_4 \to y_1 \otimes y_2 \otimes (L_0 - L_{-1})(y_3) \otimes y_4 + y_1 \otimes y_2 \otimes y_3 \otimes (L_0 - L_{-1})(y_4)$; this map lifts the connection map $\nabla_{t\partial/\partial t}$ on the space $\dot{Y}/\dot{\Delta}_4\dot{Y}$ of coinvariants

F^t (3.3) a finite set $\{a \in \mathbb{N}^I | \langle a, a+2 \rangle \leq t\}$ where t is an integer greater than (or equal to?) 1

$\mathcal{O}^t_\kappa$ (3.3) a full subcategory of $\mathcal{O}_\kappa$ whose objects have composition factors $\mathbf{L}^\kappa_a$ for some a in the set F^t; it is closed under extension and duality $V \to D(V)$

$D(V)$ (1.16) the duality functor of $\mathcal{O}_\kappa$ defined by $D(V) = {}^dV^{\#}(\infty)$, where ${}^dV = Hom_A(V, A)$; That is to say, $D(V)$ is the subspace of linear forms $V \to \mathbb{C}$ having zero values on $Q^{\#}_N V$ for some N¿1, with the inherited action of $\hat{\mathfrak{g}}$

${}^dV$ (1.16) $= Hom_A(V, A)$

$[V : \mathbf{L}^\kappa_{a'}]$ (3.9, p.924) the number of subquotients isomorphic to $\mathbf{L}^\kappa_{a'}$ in the composition series of V

$\mathbf{L}^\kappa_a$ (2.8) irreducible quotient of $\mathbf{V}^\kappa_a$, where $(a, \kappa - h)$ is the highest weight of the module

L_k (1.14, 1.15) the Sugawara operator, defined by

$$L_k(x) = \frac{1}{2\kappa} \sum_{j \geq -k/2} \sum_p (\epsilon^{-j} c_p)(\epsilon^{j+k} c_p)x + \frac{1}{2\kappa} \sum_{j < -k/2} \sum_p (\epsilon^{j+k} c_p)(\epsilon^{-j} c_p)x$$

where $\{c_p\}$ is an orthonormal basis of $\mathfrak{g}$; if it is acting on a tensor product over the set $\clubsuit$, we denote the action on the t-th factor as $L_{k;t}$; and the total action is $L_k = \sum_{t \in \clubsuit} L_{k;t}$ (1.15)

$\mathscr{C}$, $\mathscr{C}^+$, $\mathscr{C}^0$ (3.7, p.932) are abelian categories, with objects complex-vector spaces V with a given decomposition $V = \bigoplus_{\lambda \in \mathbb{C}} ({}_\lambda V)$, and $\tilde{\mathfrak{g}}$-module structure (respectively, $\tilde{\mathfrak{g}}^+$-module and $(\mathfrak{g} \oplus \mathbb{C}\mathbf{1})$-module structure for $\mathscr{C}^+$ and $\mathscr{C}^0$) such that

1. $\epsilon^n c({}_\lambda V) \subset {}_{\lambda - n}V$ for all $c \in \mathfrak{g}$ and $n \in \mathbb{Z}$ (respectively, $n \in \mathbb{N}$ and $n = 0$)
2. $\mathbf{1}$ acts as a scalar $\kappa - h$
3. the action of $\mathfrak{g}$ on each piece ${}_\lambda V$ is locally finite (i.e. has finite dimensional orbit on each vector in V)

And irreducible object in $\mathscr{C}^-$ is isomorphic to some $\mathscr{V}^\kappa_a(\lambda)$ where $\lambda \in \mathbb{C}$.

$\langle\omega c\rangle$ (4.9, p.927) $\hat{\mathfrak{g}}$ module structure on the completion $\hat{W}$ defined through the approximation property 4.9(a) of the space R of regular functions on C'

${}_\lambda V$ (2.23, p.918) subspace of vectors annihilated by some power of $(L_0 - \lambda)$; for example, we have $D(V) = \bigoplus_\lambda {}^d({}_\lambda V)$, where ${}^dV$ is defined in (1.16)

$\mathbb{N}$ in the paper, the set of natural numbers is defined as the set of non-negative integers, including zero (?)

γ_s (4.2) is a "chart": $P^1 \to C$, or, a morphism of varities from the complex projective line P^1 to a genus zero curve C with a unique marked point $p_s \in \heartsuit$ on every component, such that $\gamma_s(0) = p_s$

ν (4.16,p.931) is a $\mathbb{C}$-linear map: $T(W) \otimes X \to (W \otimes X)/G_1(W \otimes X)$, defined canonically; the image $\nu(\tilde{\mathfrak{g}}^\heartsuit(T(W) \otimes X))$ is contained in $\Gamma(W \otimes X)/G_1(W \otimes X)$ (4.17,p.932); Thus we have [1] an induced canonical map of coinvariants $(T(W) \otimes X)_{\tilde{\mathfrak{g}}^\heartsuit} \to (W \otimes X)_\Gamma$.

R (4.5, p.926) is the algebra of regular complex-valued functions on the algebraic curve C' obtained from removing points $\{p_s | s \in S(= \spadesuit \sqcup \heartsuit)\}$from the genus-zero curve C

Γ (4.6, p.926) is the central extension $\Gamma = (R \otimes \mathfrak{g}) \oplus \mathbb{C}\mathbf{1}$ of the Lie algebra of the Lie-valued regular functions on the algebraic curve C'

W (4.4, p.926) $= \bigotimes_{s\in\spadesuit} V_s$, where V_s are $\tilde{\mathfrak{g}}$ modues, with a natural filtration coming from the filtration of $U(\Gamma)$

R_n (4.8, p.927) is a subspace of $R = \mathbb{C}[C']$, consisting of those functions which have an n-th order zero (or (-n)-th order pole if n is negative) at every point $s_0 \in \heartsuit$.

G_N (4.8, p.927) the complex subspace of $U(\Gamma)$ spanned by products $(f_1c_1)\dots(f_Nc_N)$ with $f_i \in R_1$ and $c_i \in \mathfrak{g}$.

$\hat{W}$ (4.8, p.927) the projective limit of the system of vector spaces $W/G_1W \leftarrow W/G_2W \leftarrow W/G_3W \leftarrow \dots$ induced by the decreasing filtration $W \supset G_1W \supset G_2W \supset \dots$

[1] Γ being the Lie algebra $\mathfrak{g}$ tensored with the ring of regular functions over a complex curve, see p.926

Z (6.2) is the space of linear forms $Hom_{\mathbb{C}}(W, \mathbb{C})$, with the action of Γ from W

Z^N (6.2, p.934) annihilator of $G_N W$, i.e. $Z^N = \{\lambda \in Z | (\forall \xi \in G_N) \xi\lambda = 0\}$

Z^∞ (6.2) the union of all Z^N, i.e. the inductive limit of $Z^1 \subset Z^2 \subset Z^3 \subset \ldots$

ω (6.3,p.934) a collection $(\omega_s)_{s \in \heartsuit} \in \mathbb{C}((\epsilon))^\heartsuit$ of Laurent series ω_s, such that $\omega c \in \hat{\mathfrak{g}}^\heartsuit$ acts on Z^∞ via $\langle \omega c \rangle \lambda = (gc)\lambda$ where $g \in R = \mathbb{C}[C']$ is a regular function on the curve C' which expands, at each point $s \in \heartsuit$, to a series approximating ω to the N-th order, i.e. ${}^s g - \omega_s \in \epsilon^N \mathbb{C}[[\epsilon]]$ for every $s \in \heartsuit$.

$T'(W)$ (6.3, p.934) is the restriction of Z^∞ as a $\hat{\mathfrak{g}}^\heartsuit$-module to a $\tilde{\mathfrak{g}}^\heartsuit$-module; there are a $\tilde{\mathfrak{g}}^\heartsuit$ dualizing homomorphisms (6.6) $T(W)^\# \to D(T'(W))$ and $T'(W) \to T(W)^\#$ which will turn out to be isomorphisms (Remark 6.8, Theorem 7.9) under some finiteness conditions (Corollary 7.5)

$g_s(p)$ (7.2, p.938) ,for $s \in [s_0]$, $s_0 \in \heartsuit$, is the regular function in $R = \mathbb{C}[C']$ given by $g_s(p) = \frac{1}{1/\gamma_{s_0}^{-1}(p) - 1/\gamma_{s_0}^{-1}(\gamma_s(0))}$ on the s_0-component ($p \in C_{s_0}$) and $g_s = 0$ on other components ($p \in C - C_{s_0}$); These functions are introduced in the demonstration (7.3, 7.6) of the finite-dimensionality of $W/G_M W$, which in turn establishes the isomorphisms which state that $T(W)^\#$ and $T'(W)$ are dual to each other under D (Theorem 7.9, p.940). Note that after the definition at [Kazhdan-Lusztig I, p.938, 7.2(a)], the first property of the function g_s should read "${}^{s_0} g_s \in \epsilon + \epsilon^2 \mathbb{C}[[\epsilon]]$".

X_N (7.3) the complex subspace of $U(\Gamma)$ spanned by the products $(g(s_1)c_1)(g(s_2)c_2)\ldots(g(s_N)c_N)$ with $c_i \in \mathfrak{g}$ and $s_i \in \heartsuit$. We have $X_N \subset G_N$.

W_1 (7.4) $= \bigotimes_{s \in \spadesuit} V_s(N_s)$ where the $N_s \geq 1$ (7.1, p.938)are integers such that $V_s(N_s)$ are finite-dimensional and generate V_s over $\tilde{\mathfrak{g}}$ (Theorem 2.22, p.918); This vector space is introduced in the demonstration of the finite-dimensionality of the quotient $W/G_M W$.

X (Theorem 7.9) is a smooth $\tilde{\mathfrak{g}}^\heartsuit$ module where $\mathbf{1}$ acts as the scalar $\kappa - h$

B (8.1, p.942) a commutative $\mathbb{C}$-algebra with 1, as and alternative base ring to $\mathbb{C}$

Γ_B (8.2) is the B-Lie algebra $B \otimes \Gamma$, where Γ is the Lie algebra on the algebra of regular functions on the complex curve C'. There is a natural homomorphism: $\Gamma \to \Gamma_B : x \mapsto 1 \otimes_B x$

$\underline{V}_s$ (8.3) where $s \in \spadesuit$, is a collection of $\tilde{\mathfrak{g}}_B$ modules with the same central charge $\kappa - h$

$\underline{W}$ (8.3, p.943) is the tensor product $\bigotimes_{s\in\spadesuit} \underline{V}_s$ over B; It has a structure of a $\hat{\mathfrak{g}}_B$ module and hence a structure of a Γ_B module via the homomorphism $\Gamma_B \to \hat{\mathfrak{g}}_B^{\heartsuit}$ (8.2(b); c.f. 4.6(a), p.926)

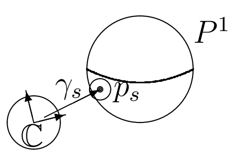
$\underline{W}$ (8.3) is the projective limit $\varprojlim_N W/G_N W$

$\underline{X}$ (8.5) is a smooth $\tilde{\mathfrak{g}}_B^{\heartsuit}$ module where $\mathbf{1}$ acts as the scalar $\kappa - h$

H (9.1) the group $PGL_2(\mathbb{C})$ of automorphisms of the projective line $P^1 = \mathbb{C} \cup \infty$

S (9.1) a finite set of at least two elements

$\mathscr{V}$ (9.1, p.950) the open subset of $H^S = \overbrace{H \times \cdots \times H}^{S}$, to be pictured as the configuration space of charts $\mathbb{C} \to P^1$ with distinct targets for their origins $0 \in \mathbb{C}$, or, configuration space of a collection S of points in P^1, each point being accompanied by a local chart



$\mathscr{V}'$ (9.1) a configuration space similar to $\mathscr{V}$, but with an extra point added, without an accompanying chart, i.e. $\mathscr{V}'$ is an open subset in $H^S \times P^1$.

$\underline{\mathscr{V}}$, $\underline{\mathscr{V}'}$ (9.2) quotients of, respectively, $\mathscr{V}$ and $\mathscr{V}'$ by the left action of $H = PGL_2(\mathbb{C})$

$A, \tilde{A}, A', \tilde{A}'$ (9.3, p.951;10.3,p.957) are, respectively, spaces of regular functions $\mathbb{C}[\underline{\mathscr{V}}], \mathbb{C}[\mathscr{V}], \mathbb{C}[\underline{\mathscr{V}'}], \mathbb{C}[\mathscr{V}']$

f_s **and** f_s^k (9.5, p.951) $f_s := \frac{1}{\gamma_s^{-1}(z)} \in \mathbb{C}$ is the reciprocal of the value of the local chart of a point z on the projective line $P(1) \cong \mathbb{C}$. The set $\{1, f_s^k(s \in S; k \geq 1)\}$ form a basis of A' over A (Lemma 9.6, p.951)

B, B' (9.7, p.952) B is a $\mathbb{C}$-algebra with a $\mathbb{C}$-homomorphism $A \to B$; $B' := B \otimes_A A'$.

Δ (9.9, p.952) is the A-Lie algebra $A' \otimes \mathfrak{g}$ with bracket $[fc, f'c'] = ff'[c, c']$. There is a natural homomorphism of A-Lie algebras $\Delta \to \hat{\mathfrak{g}}_A^S$ given by $fc \mapsto \sum_{s \in S} \delta_s({}^s fc)$

δ_s (1.6, p.908) is the Lie algebra embedding $\hat{\mathfrak{g}}_A \to \hat{\mathfrak{g}}_A^S$ extending the simple embedding $\mathfrak{g} \to \mathfrak{g}^S$

$\hat{\mathfrak{g}}^S, \tilde{\mathfrak{g}}^S$ are central extensions of $\mathfrak{g}^S = \overbrace{\mathfrak{g} \oplus \cdots \oplus \mathfrak{g}}^{S}$, i.e. they have one dimensional centres, and are not direct sums of affine Lie subalgebras

Δ_B (9.9, p.953) is the B-Lie algebra $\Delta_B = B \otimes_A \Delta (= B' \otimes \mathfrak{g})$. There is a homomorphism of B-Lie algebras $\Delta_B \to B \otimes_A \hat{\mathfrak{g}}_A^S \to \hat{\mathfrak{g}}_B^S$. There is an exact sequence (9.10(a),p.953)

$$0 \to B \otimes \mathfrak{g} \xrightarrow{\alpha} \Delta_B \oplus (\hat{\mathfrak{g}}_B^S)^+ \xrightarrow{\alpha'} \hat{\mathfrak{g}}_B^S \to 0$$

where $(\hat{\mathfrak{g}}_B^S)^+ = B[[\epsilon]]^S \otimes \mathfrak{g}$ (c.f. Lemma 9.8, 9.10)

$\underline{V}_s$ (9.11, p.953) a smooth $\tilde{\mathfrak{g}}_B$-module with central charge $\kappa - h$

Y (9.11, p.953) $= \bigotimes_{s \in S} \underline{V}_s$

Y_{Δ_B} (9.11) the space of coinvariants $Y / \Delta_B Y$

Q_N (1.7, p.908)

B_1 (9.13, p.954) another ring, for studying the base change of Y_{Δ_B}

$\mathbf{h}, \mathbf{h}^+, \mathbf{h}'\mathbf{h}'^+$ (9.14, p.955) - form a "split induction datum" (A.1, p.1008), e.g. from (9.10(a)), we can take $\mathbf{h} = (\hat{\mathfrak{g}}_B^S)^+$, $\mathbf{h}' = \Delta_B$, $\mathbf{h}'^+ = B \otimes \mathfrak{g}$. This way we can show (proposition 9.15) that the map (9.15(a)) $(B \otimes (\bigotimes_s \mathscr{N}_s))_{\mathfrak{g}} \to Y_{\Delta_B}$ is surjective and is an isomorphism; We also have the base change behaviour $(B \otimes (\bigotimes_s \mathscr{N}_s))_{\mathfrak{g}} = B \otimes \left((\bigotimes_s \mathscr{N}_s)_{\mathfrak{g}} \right)$

vector field (10.1, p.956) on a smooth variety X is an element in the Lie algebra $Der\mathbb{C}[X]$.

A_0 (10.2, p.957) $= \mathbb{C}[H] = \mathbb{C}[PGL_2(\mathbb{C})] = \mathbb{C}[GL_2(\mathbb{C})]_{\mathbb{C}^*}$, the subalgebra of $\mathbb{C}[GL_2(\mathbb{C})]$ consisting of functions invariant under the scaling action $(g_{ij} \mapsto \lambda g_{ij})$ of $\mathbb{C}^*$.

$Der(A_0)$ (10.2, p.957) The Lie algebra of derivations of A_0; It has an A_0-basis $\{\theta_0, \theta_{-1}, \theta_1\}$ given by the formulae $\theta_{-1} = \theta^{12}, \theta_0 = \theta^{11} = -\theta^{22}, \theta_1 = -\theta^{21}$ where $\frac{\partial}{\partial g_{ij}} = \frac{g_{3-i,3-i}\theta^{ij} - g_{3-i,i}\theta^{3-i,j}}{g_{11}g_{22} - g_{12}g_{21}}$

ϕ (10.3) is a map: $H \times P^1 \to P^1$ given by the evaluation $\phi(\gamma, z) = \gamma(z)$. The vector fields θ_k act on the first factor γ, and we have $\theta_k(\phi) = z^{k+1}\frac{\partial \phi}{\partial z}$

$\theta_0, \theta_{-1}, \theta_1$ (10.2- , pp.957-) vector fields on $GL_2(\mathbb{C})$, i.e. derivations of the algebra $\mathbb{C}[GL_2]$, which preserve the subalgebra $A_0 = \mathbb{C}[PGL_2]$ and hence can be regarded as vector fields on $H = PGL_2$, forming

1. an A_0-basis of $Der(A_0)$ (10.2);
2. a $\mathbb{C}$-basis of the Lie algebra of derivations of A_0 commuting with the automorphisms of A_0 induced by left translations in H (10.2);
3. a $\mathbb{C}$-basis of $\mathscr{D}_0$, (Lemma 10.6)
4. an $\tilde{A}(=\mathbb{C}[\mathscr{V}])$-basis of $Der(\tilde{A})$, (Lemma 10.6)
5. an $A(=\mathbb{C}[\underline{\mathscr{V}}])$-basis of $\mathscr{D}_1$ (Lemma 10.6)

$\mathscr{D}_0$ (p.958) it is the complex vector subspace of $Der(\tilde{A})$ generated by $\theta_{0,s}$, $\theta_{+1,s}$, $\theta_{-1,s}$ for $s \in S$ (S=the configuration of points on the Riemann sphere)

$\mathscr{D}_1$ (p.958) it is a subspace of the set of derivations, which commute with the action of H, of the algebra $\tilde{A} = \mathbb{C}[\mathscr{V}]$ on the "lifted" configuration space $\mathscr{V}$ of marked points on the Riemann sphere, so that it maps surjectively $\mathcal{D}_1 \to Der(A)$ onto the space of derivations on the moduli space $\underline{\mathscr{V}}$

$\mathscr{D}_2$ (10.12, p.961) it is the Lie subalgebra of $\mathscr{D}_1$ consisting of derivations sending the subalgebra $A \subset \tilde{A}$ to zero, i.e. they are derivations along the fibres of the projection $\mathscr{V} \to \underline{\mathscr{V}}$; It is also an A-Lie algebra: the Lie algebra structure and the A-module structure on $\mathscr{D}_2$ are compatible. We have (12.11, p.969) $Der(A) = \mathscr{D}_1/\mathscr{D}_2 \to End_{\mathbb{C}}(Y/\Delta Y)$.

$p_{n,s'}$ (10.8) n-th coefficient of the series expansion of a function $f \in A'$ at a point $\gamma_{s'}(0)$

$q_{n,s'}$ (10.8) n-th coefficient of the series expansion of $\theta_{k,s} f$ at a point $\gamma_{s'}(0)$.

$l_{ss'}, m_{ss'}, k_{ss'}$ (10.10) they are elements in A; $l_{ss'}, m_{ss'}$ being the coefficients in the expansion ${}^{s'}f_s = l_{ss'} + m_{ss'}\epsilon + \cdots \in A[[\epsilon]]$, where $s \neq s'$; $k_{ss'} \in A'$ is defined by $f_s f_{s'} = l_{ss'} f_{s'} + l_{s's} f_s + k_{ss'}\mathbf{1}$. In fact, $k_{ss'}$ is constant along any fibre of $\underline{\mathscr{V}}' \to \underline{\mathscr{V}}$ and so $k_{ss'} \in A$.

ξ (10.13) is an element $\xi = \sum_{j,s} a_{j,s}\theta_{j,s} \in \mathscr{D}_2$; $a_{j,s} \in A$. The coefficients $a_{0,s}, a_{-1,s}$ satisfy a relation with the coefficients $l_{ss'}$.

σ_k (10.14) where $k = 0, +1, -1$, is a vector field on H^S obtained from θ_{k,s_0} through left translation by $\gamma_{s_0}^{-1}$ for every H-factor for which $s \neq s_0$, where s_0 has been fixed from the beginnning. We have $\sigma_k \in \mathscr{D}_2$. They form an A-basis for $\mathscr{D}_2$.

$\Lambda_{\theta_{k,s}}$ (11.2) where $k = 0, +1, -1$; $s \in S$ is defined to be $1 \otimes \cdots \otimes L_{k;t} \otimes \cdots \otimes 1$; where $L_{k;t}$ is the Sugawara opeartor L_k acting on $\underline{V}_s$ (p.953) at the s-th position in $Y = \bigotimes_{s\in S} \underline{V}_s$

$\mathfrak{g}'$ (11.1) $= \mathfrak{g}^{\clubsuit}$; $\clubsuit$ is a finite set

Λ (11.2) is a complex-linear map: $\mathscr{D}_0 \to End_B(Y) : \partial \mapsto \Lambda_\partial$. It is actually (11.2(a)) a homomorphism of Lie algebras over $\mathbb{C}$.

Λ', Λ'' (11.3) $\Lambda' : \mathscr{D}_1 \to End_B(Y)$ is the unique A-linear extension of Λ to $\mathscr{D}_1$; Λ'' is the restriction of Λ' to $\mathscr{D}_2$. Λ'' is a homomorphism of A-Lie algebras.

$\Theta_{k,s,\sigma}$ (11.5) is a B-linear map: $Y \to Y$ defined as

$$1 \otimes 1 \cdots \otimes \underbrace{\left(\sum_n \theta_{k,s}(p_{n,\sigma})\epsilon^n \hat{c} \right)}_{\sigma-factor} \otimes \ldots 1 \otimes 1$$

$r_{s,s'}$ (Lemma 11.9)

∇ (12.2, p.967) a connection on $\mathscr{A}$-module M, i.e. a $\mathbb{C}$-linear map $\nabla : M \to \Omega^1_{\mathscr{A}} \otimes_{\mathscr{A}} M$, satisfying the Leibniz condition $\nabla(am) = d(a) \otimes m + a\nabla(m)$ for $a \in \mathscr{A}, m \in M$; where $\Omega^1_{\mathscr{A}} = \mathscr{I}/\mathscr{I}^2$ is an $\mathscr{A}$-module and $\mathscr{I}$ is the kernal of the multiplication map $\mathscr{A} \otimes \mathscr{A} \to \mathscr{A}$. There is an isomorphism $Hom_{\mathscr{A}}(\Omega^1_{\mathscr{A}}, \mathscr{A}) \cong Der(\mathscr{A})$ given by $f \mapsto fd$.

∇_∂ (12.3) is a map in $End_{\mathbb{C}}(M)$; given a derivation $\partial \in Der(\mathscr{A})$, ∇_∂ is given by the composition of ∇ with $\partial \otimes 1_M$:

$$M \to \Omega^1_{\mathscr{A}} \otimes_{\mathscr{A}} M \to \mathscr{A} \otimes_{\mathscr{A}} \otimes M \cong M$$

$\nabla_\partial(m)$ is $\mathscr{A}$ linear in the two variables m and ∂.

∇^0, ∇^1 (12.7, p.968) they are connections on the A-module Y, with $\nabla^0_\partial(fy) = \partial(f)y$ and $\nabla^1_\partial = \nabla^0 \pi(\partial) + \Lambda'_\partial$, where $\pi : \mathscr{D}_1 \to Der A$ is the canonical map. $\nabla^1 : \mathscr{D}_1 \to End_{\mathbb{C}}(Y)$ is a homomorphism of Lie algebras over $\mathbb{C}$.

$\underline{\mathscr{V}}_0$ (13.1) is a contractible real analytic subset of $\underline{\mathscr{V}}$

$\tau, \bar{\tau}$ (14.1, p.974) which form parts of the commutativity isomorphisms in the category, are maps $V \to V$, given by $\mathscr{O}_\kappa$ given by $\tau = e^{i\pi L_0} e^{L_1}; \bar{\tau} = e^{-i\pi L_0} e^{L_1}$

$P, \bar{P}$ (14.2) are maps: $V \otimes V' \otimes V'' \to V' \otimes V \otimes V''$ given by $P(x \otimes y \otimes z) = \tau y \otimes \tau x \otimes \bar{\tau} z$ and $\bar{P}(x \otimes y \otimes z) = \bar{\tau} y \otimes \bar{\tau} x \otimes \tau z$. There are induced isomorphisms of coinvariants (14.5) $(V \otimes V' \otimes V'')_{\Delta_{\mathbb{C}}} \to (V' \otimes V \otimes V'')_{\Delta_{\mathbb{C}}}$ where $\mathbb{C}$ is an A-algebra via evaluation at a point.

$\mathscr{P}, \bar{\mathscr{P}}$ (14.6(d)) is the transpose $(V \dot{\otimes} V') \to (V' \dot{\otimes} V)$ in $\mathscr{O}_\kappa$ of the map $D(V' \dot{\otimes} V) \cong D(V \dot{\otimes} V')$ constructed at (14.6(b)). The two maps $\mathscr{P}, \bar{\mathscr{P}}$ are in fact inverse of each other (p.977).

Φ_n (section 15, p.977; 15.24, p.983; 15.25) where $n \geq 1$, is a family of maps relating the two spaces of coinvariants on non-singuar and degenerate quadrics (i.e. union of two lines). Φ_n is constructed as a B_n-linear map (15.25(a), p.984): $T(W_n)^\#/\Delta_{2,n} T(W_n)^\# \to W_n/\Delta_{4,n} W_n$, where $1 \leq n < \infty$. It is shown in §16 that Φ_n is an isomorphism for every $n \geq 1$. (proposition 15.27, p.985)

Ψ_n (15.20, p.982) is an isomorphism of B_n-modules (15.20(a)):

$$\left(B_n \otimes (V_1 \dot{\otimes} V_2) \otimes (V_3 \dot{\otimes} V_4)\right)_{\Delta_{2,n}} \cong T(W_n)^\#_{\Delta_{2,n}}$$

Y_n (15.10, p.979) $= A_n \otimes \left(\bigotimes_{i=1}^4 V_i\right)$, a $\hat{\mathfrak{g}}_{A_n}^S$-module and hence a $\Delta_{4,n}$-module

B_∞, B'_∞ (15.13, p.980) $B_\infty = \mathbb{C}[t]$ and $B'_\infty = \mathbb{C}[t,p,q]/(pq-t)$

B_n, B'_n (15.13) $B_n = B_\infty/(t^n)$, and $B'_n = B'_\infty/(t^n)$

X_{12}, X_{34} are tensor products $V_1 \dot{\otimes} V_2$ and $V_3 \dot{\otimes} V_4$ respectively

[12] is the set {1,2,12}.

$\gamma_1, \gamma_2, \gamma_{12}, \gamma_3, \gamma_4, \gamma_{34}$ are charts on the complex curve $C = P^1 \sqcup P^1$ sending the points $\{0,1,\infty\}$ on $\mathbb{C}$ to copies of such on the curve; e.g. $\gamma_1 : P^1 \cong C_{12} : z \mapsto \frac{1}{1-z}$; and also $\gamma_{12} : z \mapsto z$.

R (15.18, see also 4.5, p.926) algebra of regular functions on the curve with points removed; the case of §15.18, where the curve $C = P^1 \sqcup P^1$ and $S = \{1,2,12,3,4,34\}$, we have $R = \mathbb{C}[u, u^{-1}, (1-u)^{-1}] \oplus \mathbb{C}[v, v^{-1}, (1-v)^{-1}]$

$W_n, \hat{W}_n, \hat{W}$ (15.19) for $W = \bigotimes_{i=1}^4 V_i$, we have: $W_n = B_n \otimes W$; and $\hat{W}_n = \varprojlim_N W_N / G_N W_n$; and $\hat{W} = \varprojlim_N W/G_N W$

$T(f,c), T'(f,c)$ (lemma 15.23) with $f \in \mathbb{C}[u, (1-u)^{-1}$ and $c \in \mathfrak{g}$, $T(f,c)$ and $T'(f,c)$ are two B_n-linear maps: $W_n \to W_n$

t, p, q, u, v (16.1) are variables

T (16.2, p.985) is the subalgebra $B_n[u^{-1}] \oplus B_n[v^{-1}]v^{-1}$ of the algebra $B[u, u^{-1}] \oplus B[v, v^{-1}]$.

$\mathscr{C}$ is the ring of functions $\frac{B_n[p,q]}{(pq-t)}[(p-1)^{-1}, (q-1)^{-1}]$

$P_0, P, \hat{P}_0, \hat{P}$ (16.9) are Lie algebras with coefficients being rational functions or formal power series; e.g. $P = P_0 + B_n \mathbf{1}$ where $P_0 \cong \frac{B_n[p,q]}{pq-t} \otimes \mathfrak{g} \subset \tilde{\mathfrak{g}}_{B_n}^{\heartsuit}$

X_n (16.10, p.989) Beilinson's "diagonal module" $X_n = U(\tilde{\mathfrak{g}}_{B_n}^{\heartsuit}) \otimes_{U(P)} B_n$

Q (16.11) a Lie subalgebra of $\tilde{\mathfrak{g}}_{B_n}^{\heartsuit}$, in which it is the complement of P.

Π_n (16.17, p.992; 17.23,p.1003) (see 8.5(a), p.994) a map from $\tilde{\mathfrak{g}}_{B_n}^{\heartsuit}$-coinvarinats to Γ-coinvariants. It is compatible (17.23) with the connections $\nabla_{t\partial/\partial t}$.

ν (17.1) a morphism: $\dot{C} \to \underline{\mathscr{V}}$

$\mathscr{H}$ (17.8, p.995) a linear map on the tensor product $\bigotimes_i V_i$ defined by the Sugawara operators: $y_1 \otimes y_2 \otimes y_3 \otimes y_4 \to y_1 \otimes y_2 \otimes (L_0 - L_{-1})(y_3) \otimes y_4 + y_1 \otimes y_2 \otimes y_3 \otimes (L_0 - L_{-1})(y_4)$; this map lifts the connection map $\nabla_{t\partial/\partial t}$ on the space $\dot{Y}/\dot{\Delta}_4\dot{Y}$ of coinvariants

$\mathscr{M}$, $\dot{\mathscr{M}}$ (17.10) $\mathscr{M} = Y_\infty/\Delta_{4,\infty}Y_\infty$, $\dot{\mathscr{M}} = \dot{Y}/\Delta_4\dot{Y}$. It is an A_∞-module and has a natural connection with regular singularity at 0 (17.14, p.998).

∇, ∇' (17.16) are $\mathbb{C}$-linear maps: $\mathbb{C}[t] \otimes X_{12} \otimes X_{34} \to \mathbb{C}[t] \otimes X_{12} \otimes X_{34}$ defined by the Sugawara operator $L(0)$. They induce the same complex-linear map $\nabla_{t\partial/\partial t}$ on the space of coinvariants $(\mathbb{C}[t] \otimes X_{12} \otimes X_{34})_{\Delta_{2,\infty}}$, and on this $\mathbb{C}[t]$-module it defines a connection with regular singularity

$j(w)$ (17.23, p.1003) image of an element $w \in T(W)^\#$ in $T(W_n)^\#$, w being represented as a sequence $(w_1, w_2, \dots)$ in $hatW$

$\tau(M)$ (17.24, p.1004) torsion module of M over A_∞, i.e. vectors annihilated by powers of t.

M^f (17.24) $= M/\tau(M)$

$\hat{M}$ (17.24) $= \varprojlim_{n \geq 1} M/t^n M$

$\hat{\mathbf{M}}$ (17.24, p.1005) $= Hom_{\mathbb{C}[[t]]}\left(\hat{\mathscr{M}}'^f, \hat{\mathscr{M}}^f\right)$

$\mathscr{M}_{an}, \mathscr{M}'_{an}, \mathbf{M}_{an}$ (17.26, p.1005) real analytic bundles on the real interval $(-\infty, 1)$

ω_n (17.27(a)) an element in $\mathbf{M}/t^n\mathbf{M}$ constructed from an isomorphism of quotients of $\mathscr{M}$ and $\mathscr{M}'$.

$\tilde{\omega}$ (17.27, 17.28) an analytic section of $\mathbf{M}_{an}$, horizontal with respect to the connection $\nabla_{t\partial/\partial t}$, which is the analytic continuation along the interval $(-\infty, 1)$ of the power series ω at 0,

I (19.1) a finite set, to be the set of indices of simple roots

(a_{ij}) (19.1) a Cartan matrix

(b_{ij}) (19.1) inverse of Cartan matrix

δ_i (19.1) symmetrizer, i.e. $(\delta_i a_{ij})$ is a symmetric matrix, it is assumed to be positive-definite

ω_i (19.1) fundamental weights

α_i (19.1) simple roots

$\langle\rangle$ (19.1) pairing in the weight lattice: $\langle a, b\rangle = \sum_{i,j} \delta_i b_{ij} a(i) b(j)$

W (19.1) the Weyl group, i.e. the finite subgroup of $Aut(\mathbb{Z}^I)$ generated by the reflections $s_i : s_i(\omega_j) = \omega_j - \delta_{ij}\alpha_i$ for $j \in I$.

w_0 (19.1) longest element of the Weyl group; $w_0(\omega_i) = -\omega_{\bar{i}}$ for every $i \in I$.

$\mathfrak{g}$ (p.335, 19.1) a simple Lie algebra with (a_{ij}) as Cartan matrix

ϖ (§19) a formal variable, to be thought of as $1/\kappa$.

A (19.1, p.336) $= \mathbb{C}[[\varpi]]$

$\mathfrak{A}$ (19.1) the universal envelope of $\mathfrak{g}$

v (19.1) $= e^{i\pi\varpi} \in A$

v_i (19.1) $= v^{\delta_i}$

$[n]_i$ (19.1) $= \frac{v_i^n - v_i^{-n}}{v_i - v_i^{-1}}$

t (19.2) is an element in $\mathfrak{g} \otimes \mathfrak{g}$ which commutes with every Lie-like element $(1 \otimes x + x \otimes 1)$; Under the multiplication map it is mapped into the quadratic Casimir element $\Omega \in \mathfrak{A}$; we have $t = (1/2)(\Delta(\Omega) - 1 \otimes \Omega - \Omega \otimes 1)$

V_a (19.3, p.337) finite dimensional irreducible module of highest weight a

y_a (19.3) highest weight vector of V_a; we have $e_i y_a = 0, h_i y_a = a(i) y_a$.

$\mathscr{D}$ (§19, see 19.3,p.337) Drinfeld's category, with as objects free $\mathfrak{A}$-modules of finite rank over $\mathbf{A}$; and as morphisms $\mathfrak{A}$-linear maps. Its indecomposable objects are V_a where a is in the weight lattice (19.3). There is a structure of tensor category on $\mathscr{D}$ with commutativity (19.12) and associativity (19.10) isomorphisms constructed from (formal) Knizhnik-Zamolodchikov equations (19.8(a)) - see $R(\Pi_0, \Pi_1), or A_{V_1,V_2,V_3}(19.10)$.

θ, θ_i (19.4) is a map: $V \to V$ defined, for $x \in V^\lambda$, as

$$\theta_i(x) = \sum_{p,q,r \in \mathbb{N}; p-q+r=\lambda(i)} \frac{(-1)^q}{p!q!r!} f_i^p e_i^q f_i^r x$$

Given a choice of reduced expression of $w_0 = s_{i_1} \dots s_{i_N}$, we can define the operator $\theta = \theta_{i_1} \dots \theta_{i_N}$ which is an isomorphism of V, independent of the choice of reduced expressions. We also have $\theta(f_i x) = -e_{\bar{i}} \theta(x)$. θ acts on a tensor product as $\theta \otimes \theta$.

x_a (19.4) $= \theta(y_a)$. It satisfies relations $f_i(x_a) = 0$ and $h_i(x_a) = -\bar{a}(i) x_a$.

${}^0V$ (19.8) a vector subspace in V such that $V = {}^0V \otimes_{\mathbb{C}} \mathbf{A}$.

Π_0, Π_1 (19.8, 19.9, 19.10) endomorphisms of V, in particular, the quadratic commuting element t acting on the 12- or 23- components of $V_1 \otimes V_2 \otimes V_3$.

$\mathscr{X}$ (19.8, p.338) space of analytic (in the sense of §19.8, p.338) solutions on the interval $(0, 1)$ with values in vector space V of the (formal KZ) equation $\frac{df}{dz} = \frac{\varpi \Pi_0 f}{z} + \frac{\varpi \Pi_1 f}{z-1}$ (19.8(a)).

$\tilde{f}(z)$ (19.8) is the rescaled function $z^{\varpi \Pi_0} f(z)$ where f(z) is a solution of the formal KZ equation (19.8(a)).

$\tilde{f}_n$ (19.8, p.339) Fourier coefficients of the series expansion of $\tilde{f} = \sum_{n \geq 0} \varpi^n \tilde{f}_n$. It is a function: $(0, 1) \to {}^0V$ which extends to the interval $[0, 1)$ (p.339).

$R_0(f), R_1(f)$ (p.339) are two maps from the space $\mathscr{X}$ of solutions of KZ equations (19.8(a)) to the representation space V. $R_0(f)$ is the rescaled asymtotic of f at 0, i.e.

$$R_0(f) = \lim_{z \to 0} \tilde{f} = \sum_{n \geq 0} \varpi^n \lim_{z \to 0} \tilde{f}_n(z) \in V.$$

$R_1(f)$ is a similar asymptotic at 1, where f is rescaled by $(1 - z)^{-\varpi \Pi_1}$.

$R(\Pi_0, \Pi_1)$ (p.339) is the isomorphism $R_1 \cdot R_0^{-1} : V \to \mathscr{X} \to V$

A_{V_1,V_2,V_3} (19.10, p.340, 339) the associativity isomorphism

$$R(t_{12}, t_{23}) : V_1 \otimes V_2 \otimes V_3 \to V_1 \otimes V_2 \otimes V_3$$

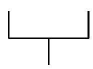
B_{V_1,V_2,V_3,V_4} **(or B')** (19.10, p.340) are maps from $(V_1 \otimes V_2) \otimes (V_3 \otimes V_4)$ to $V_1 \otimes (V_2 \otimes V_3) \otimes V_4$; with the target tensor product associated in the two possible ways.

C_{V_1,V_2} (19.12) the commutativity isomorphism: $C_{V_1,V_2}(x \otimes y) = e^{i\pi\varpi t}(y \otimes x)$, where the exponential is defined as a Taylor series.

Pentagon identity (19.11) compatibility of associativity isomorphism with itself in a quadruple tensor product

Hexagon identities (19.13) the two compatibilities of associativity with commutativity in a triple tensor prodcut. They provide solutions for the Yang-Baxter equation.

$T_{a,b}$ (20.1): the unique morphism $V_{a+b} \to V_a \otimes V_b$ of modules over $\mathfrak{g}$, mapping the highest weight vector to the highest weight vector

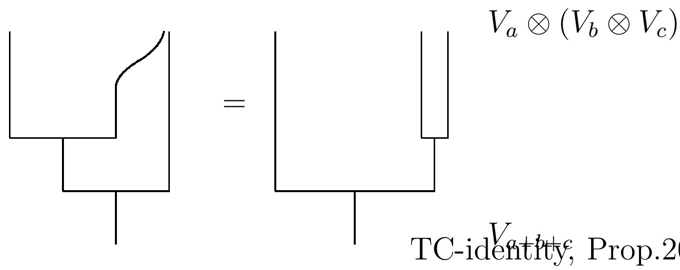


S'_c (20.4): a morphism: $V_{\bar{c}} \otimes V_c \to V_0 \simeq A =$ the ring of formal power series over a variable (which will turn out to be $1/(k + \check{h})$, the level plus the dual coxeter number), whose value at the highest weight vector $x_{\bar{c}} \otimes y_c$ is normalised as 1

S_c (20.12): the morphism S'_c normalised by an "admissible collection" (20.11) of coefficients $\{g_a\}$ satisfying some cocycle conditions with the set $\{g_{a,b}\}$ (20.6(a))

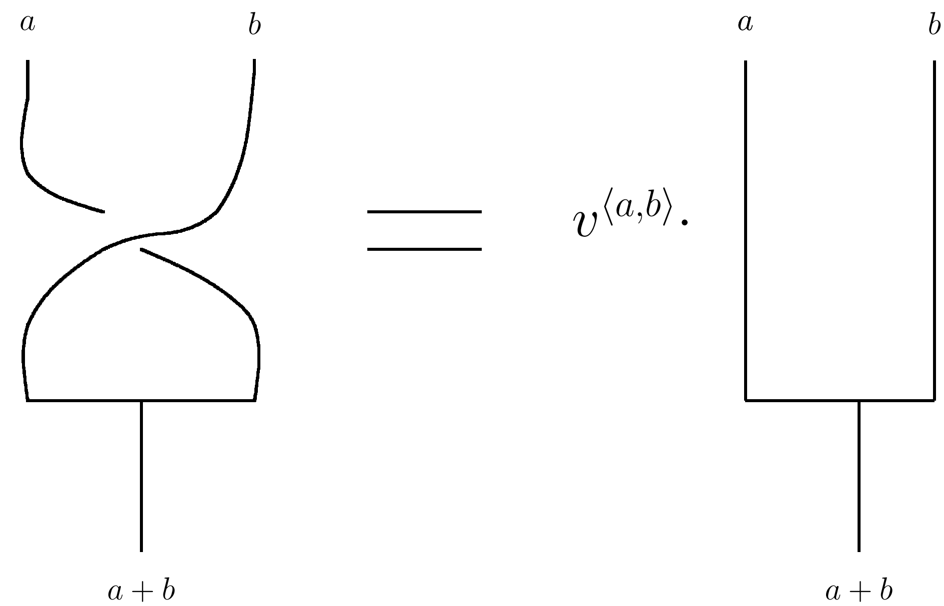
$g_{a,b}$ (20.6(a)): elements in $\mathbf{A}$, measuring the discrepancy of the S'_c morphism at different weight c's before and after the action of the morphism $T_{a,b}$

TT-identity (20.2) the associativity of the morphism $T_{a,b}$ with itself in an iteration

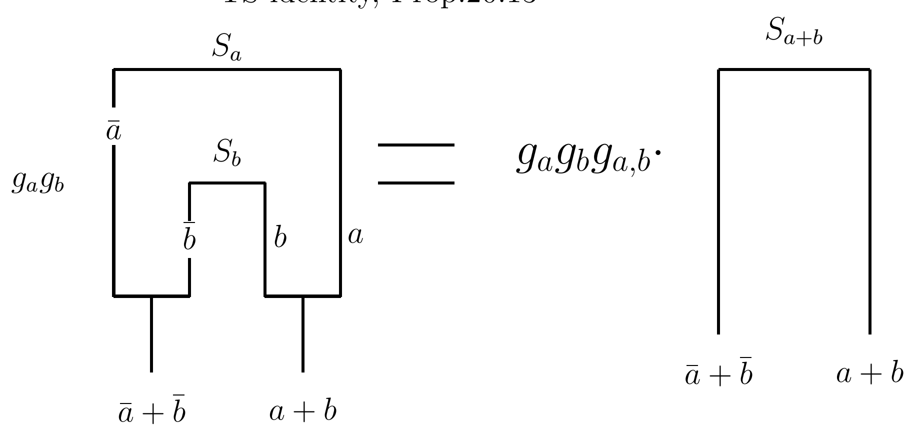


TC-identity, Prop.20.3

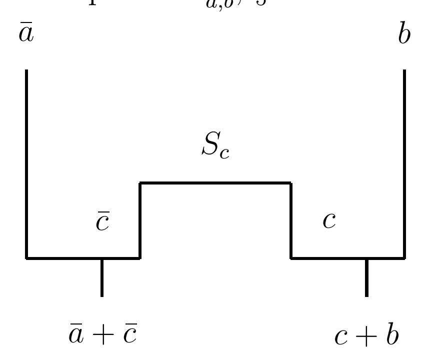
TC**-identity** (20.3) the compatibility between the T morphism of tensor products $V_{a+b} \to V_a \otimes V_b$ and the commutativity isomorphism C_{V_1,V_2}.



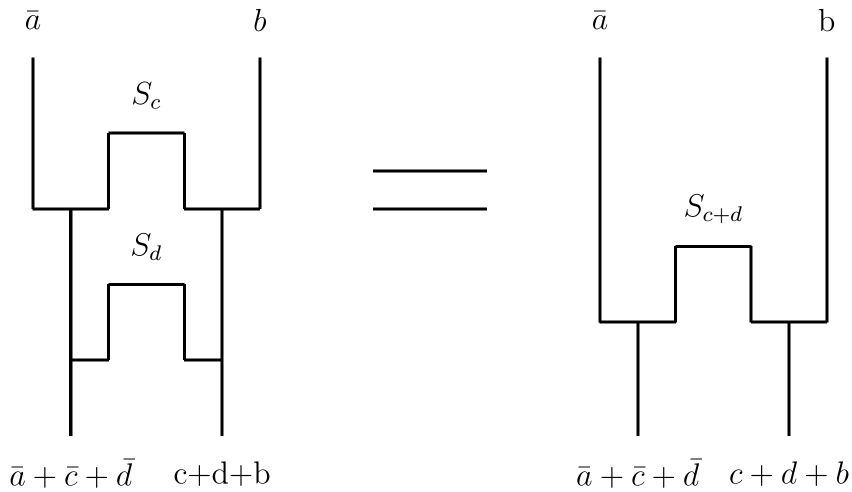
TS-identity, Prop.20.13



Morphism $tr^c_{a,b}$, §21.1

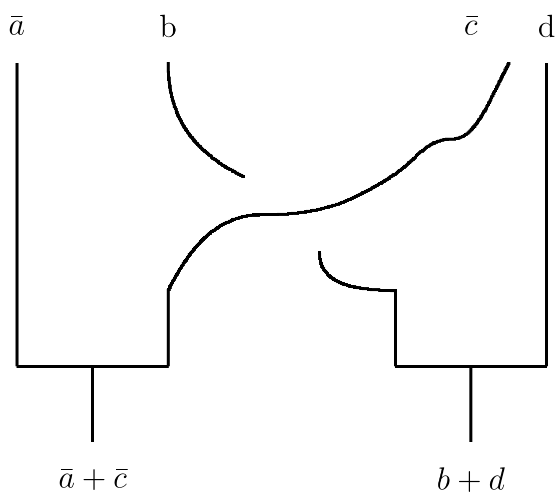


Transitivity of tr, Prop.21.3

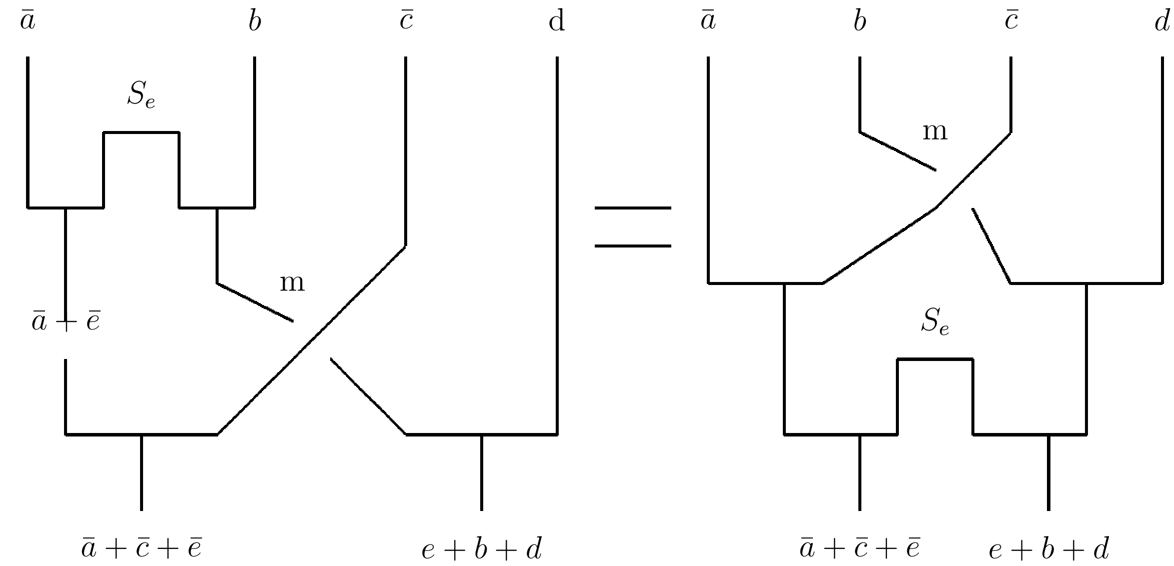


m-morphism (§21.4)

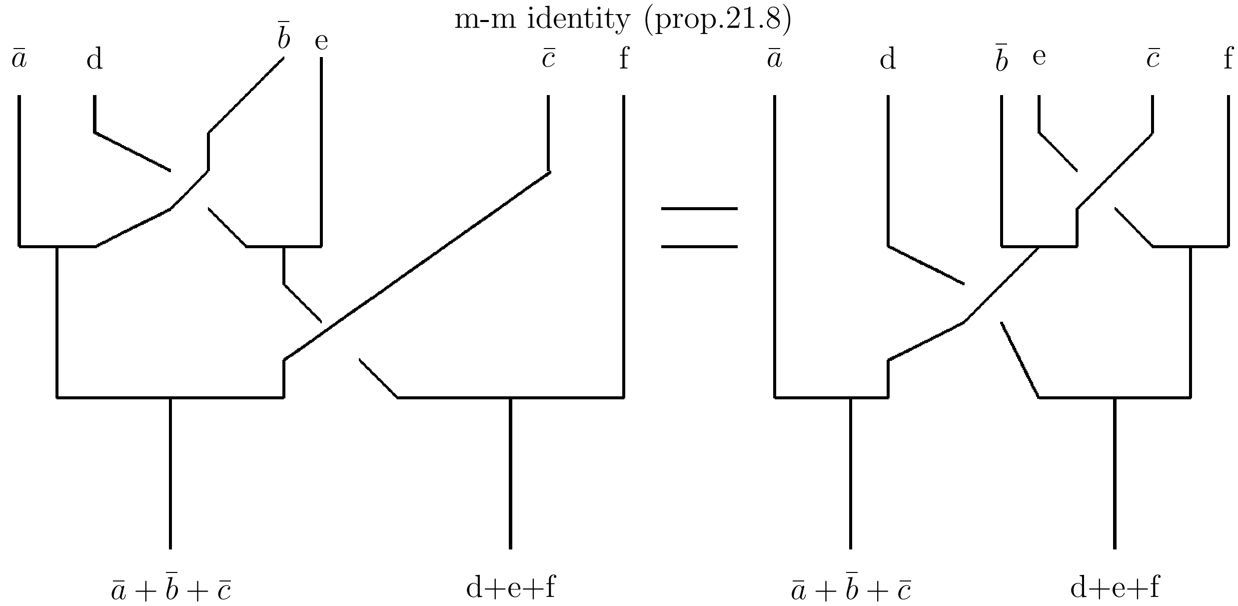
$v^{\langle b,c \rangle}$.

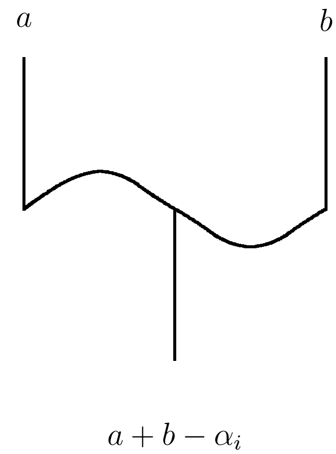


Left compatibility of m- and tr-morphisms, Prop.21.6



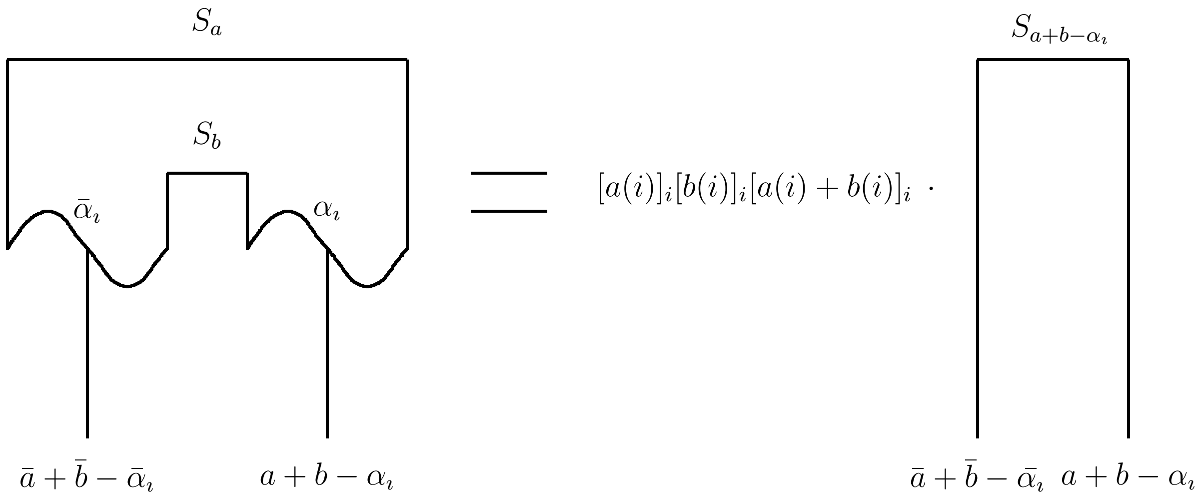
m-m identity (prop.21.8)



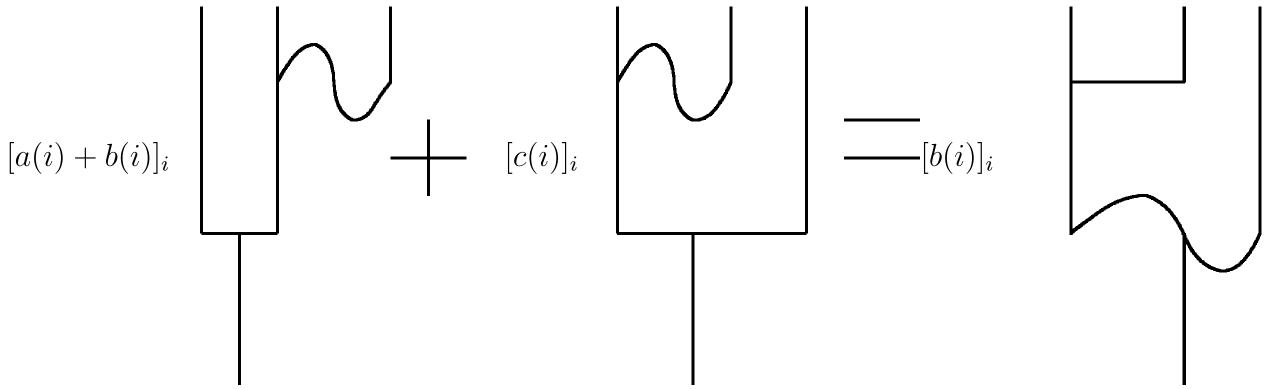


$\tau'_{i;a,b} : V_{a+b-\alpha_i} \to V_a \otimes V_b$ (22.1)

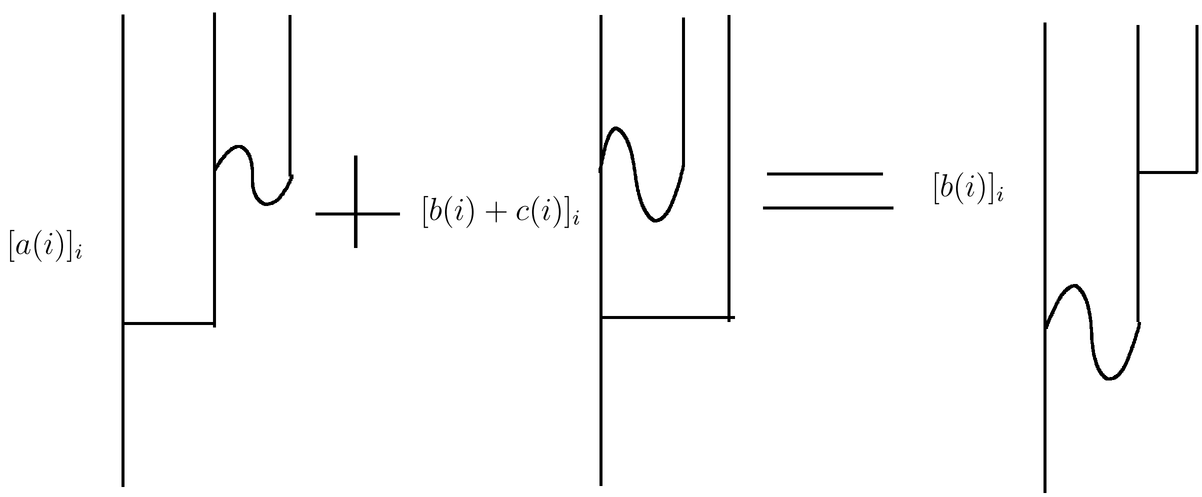
$\tau - S$-identity, §23.7



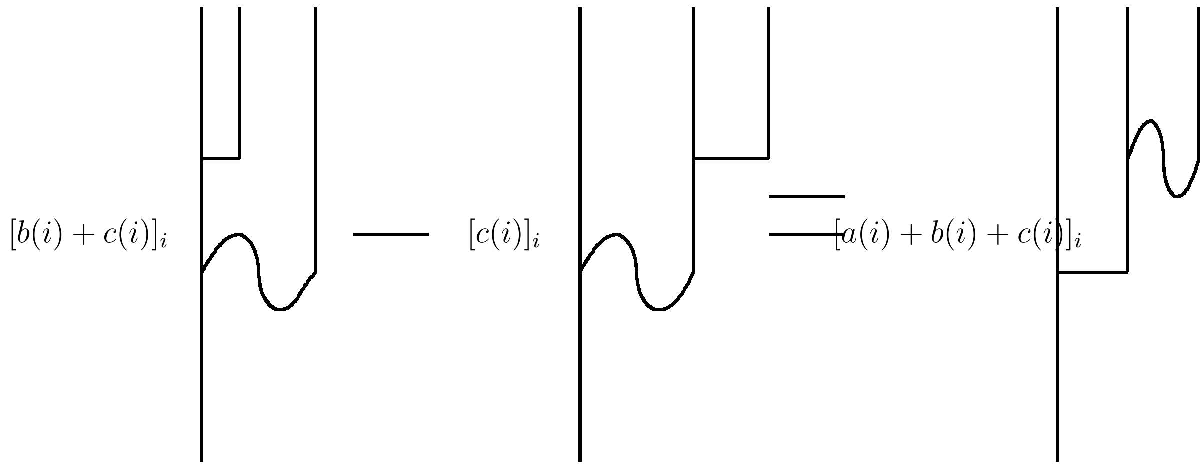
τT-identity (1), Prop.22.6



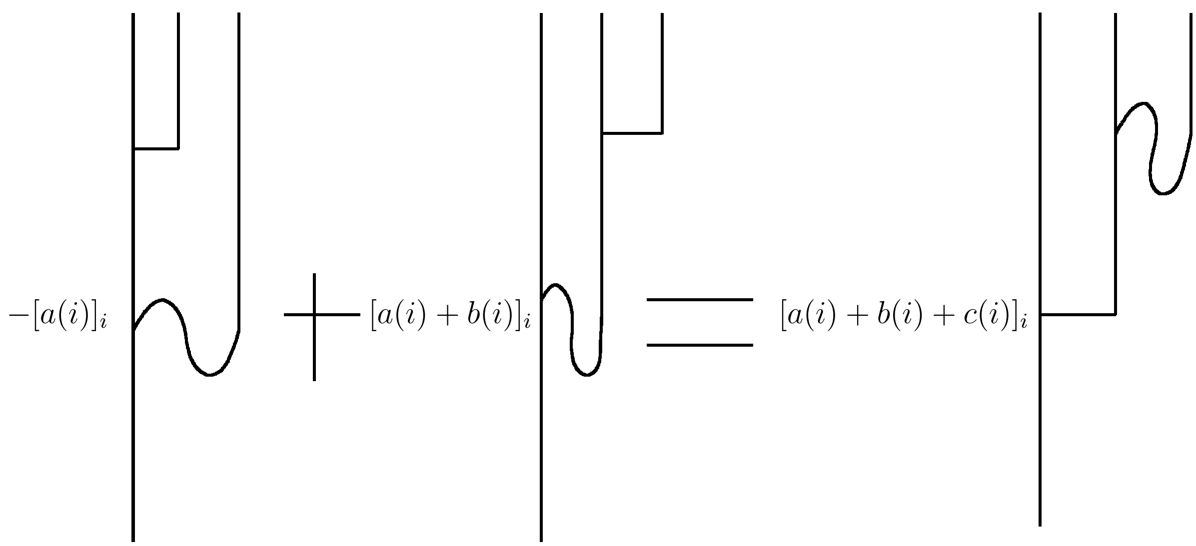
$\tau - T$-identity (2), Prop.22.6



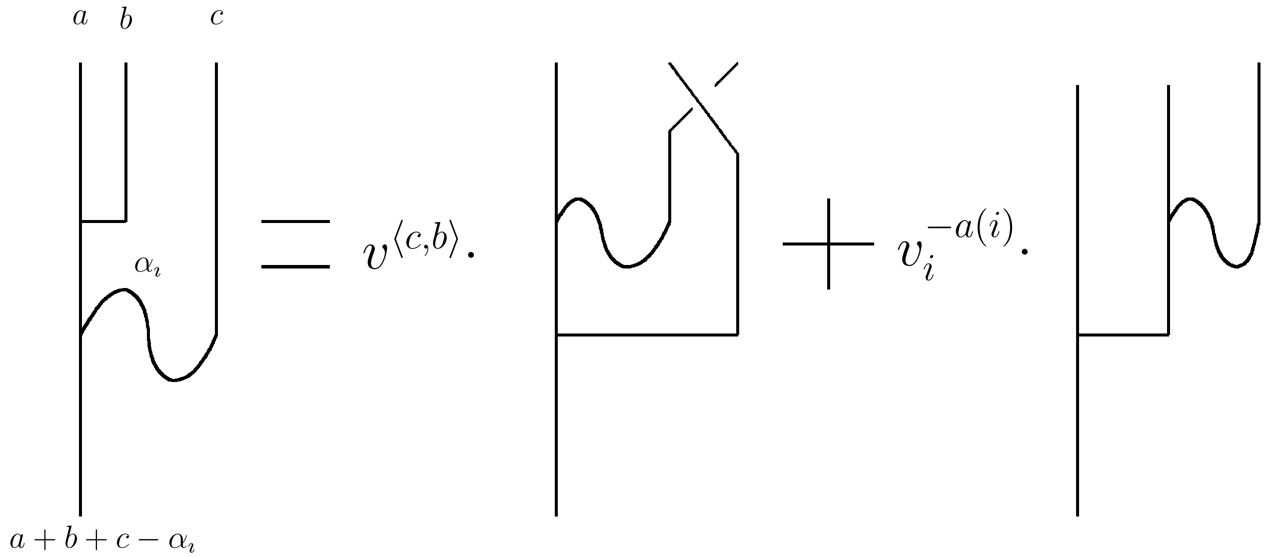
$\tau - T$-identity (3), Prop.22.6



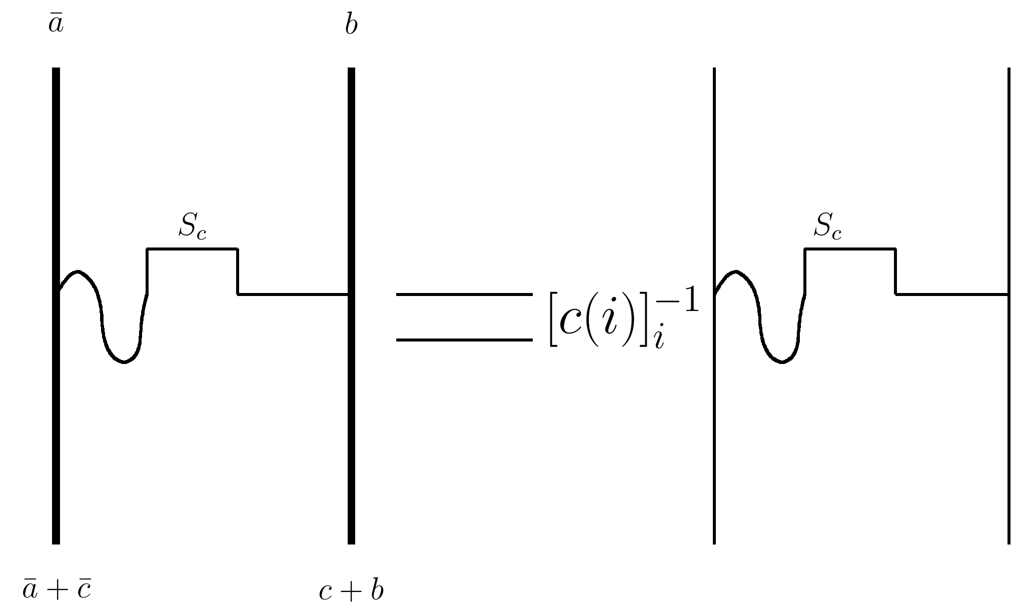
$\tau - T$-identity (4), Prop. 22.6



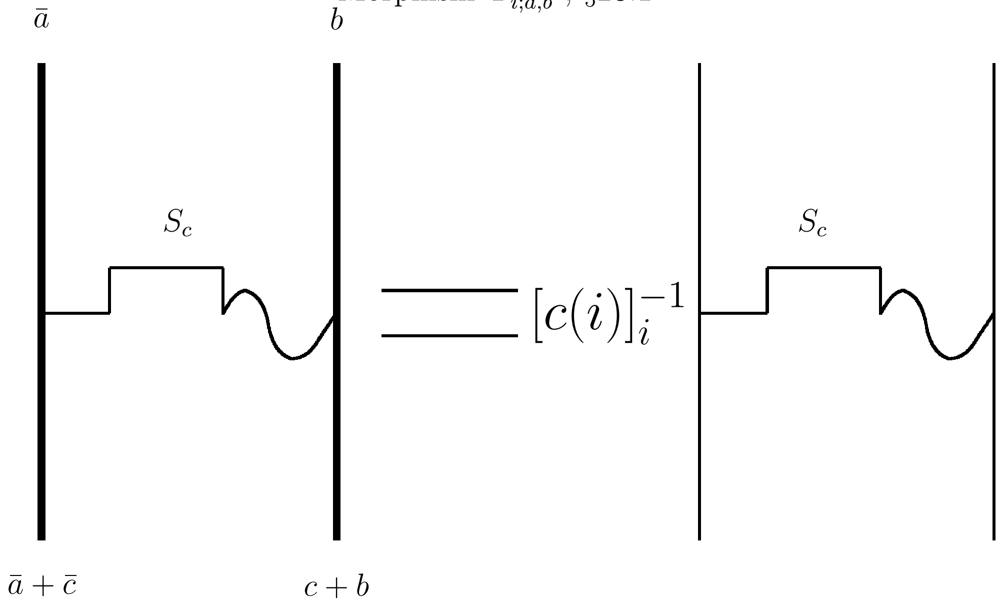
A relation for τ, T and C , Cor. 22.9



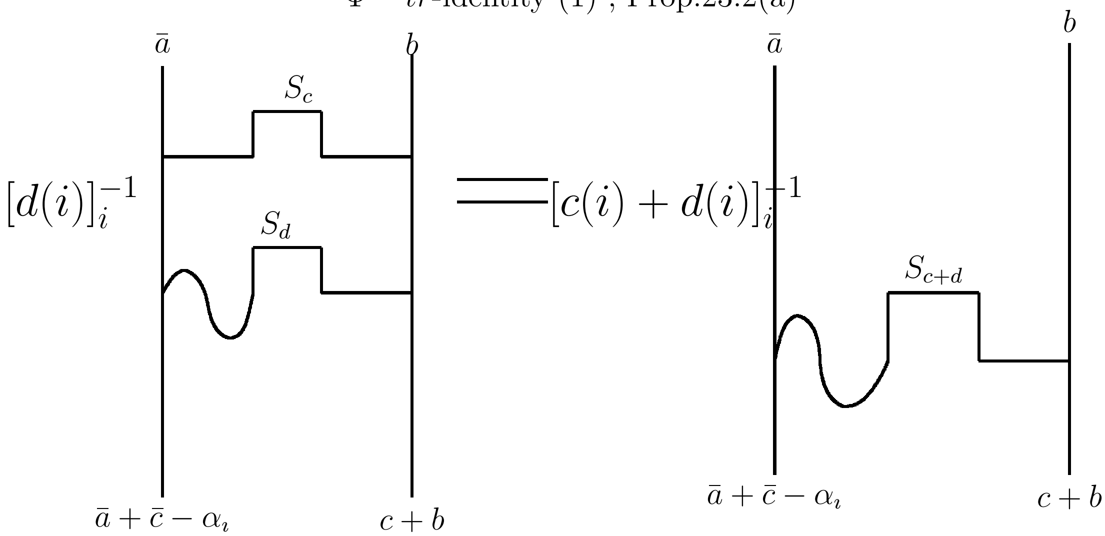
Morphism $\Phi^c_{i;a,b}$, §23.1



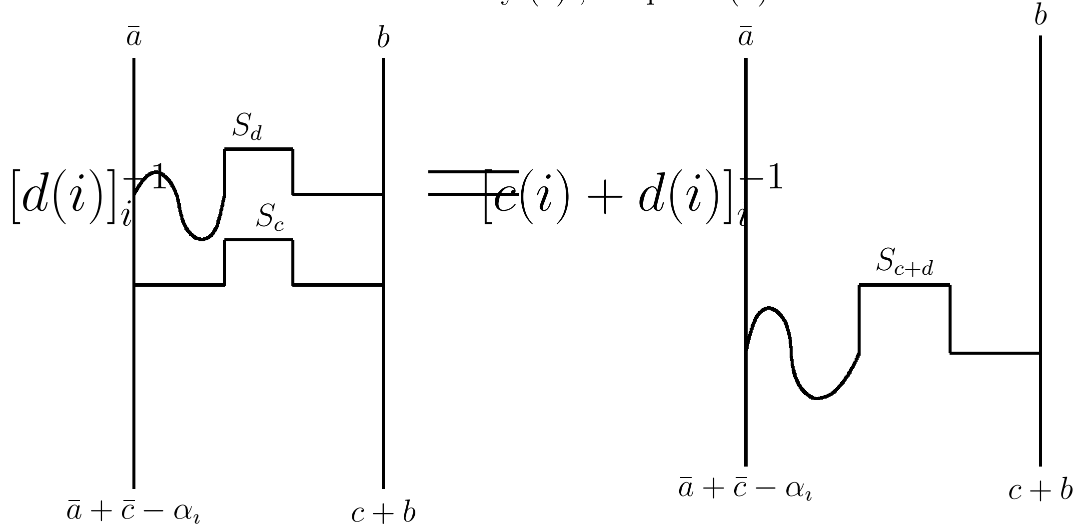
Morphism $\Psi^{c}_{i;a,b}$, §23.1



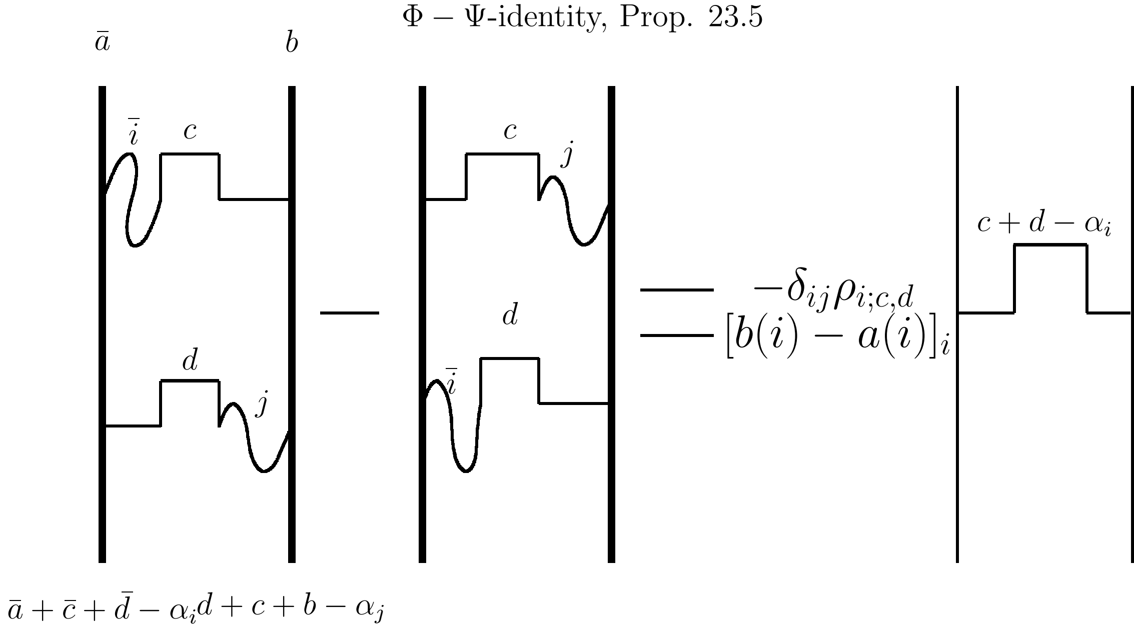
$\Phi - tr$-identity (1) , Prop.23.2(a)



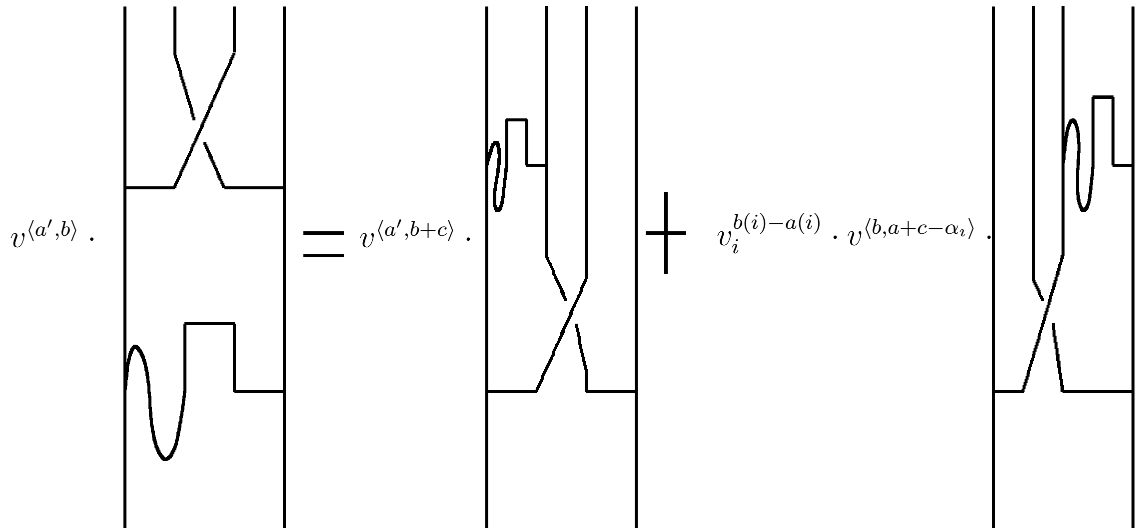
$\Phi - tr$-identity (2) , Prop.23.2(b)



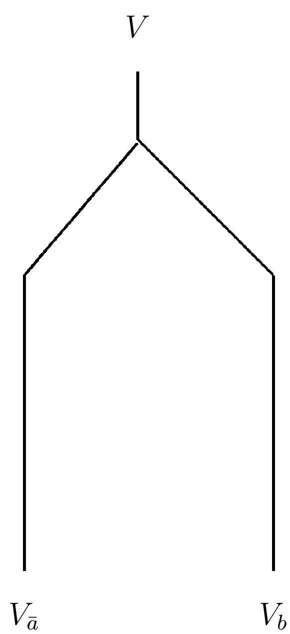
$\Phi - \Psi$-identity, Prop. 23.5

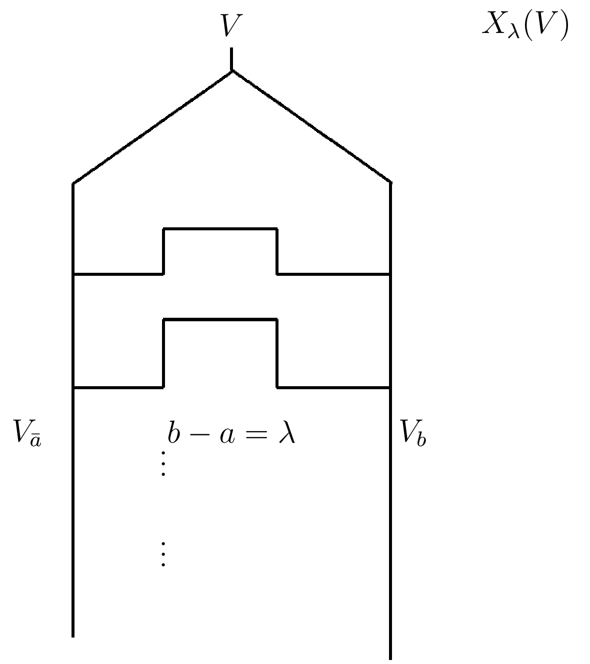


$\Phi - m$-identity, Prop. 23.9

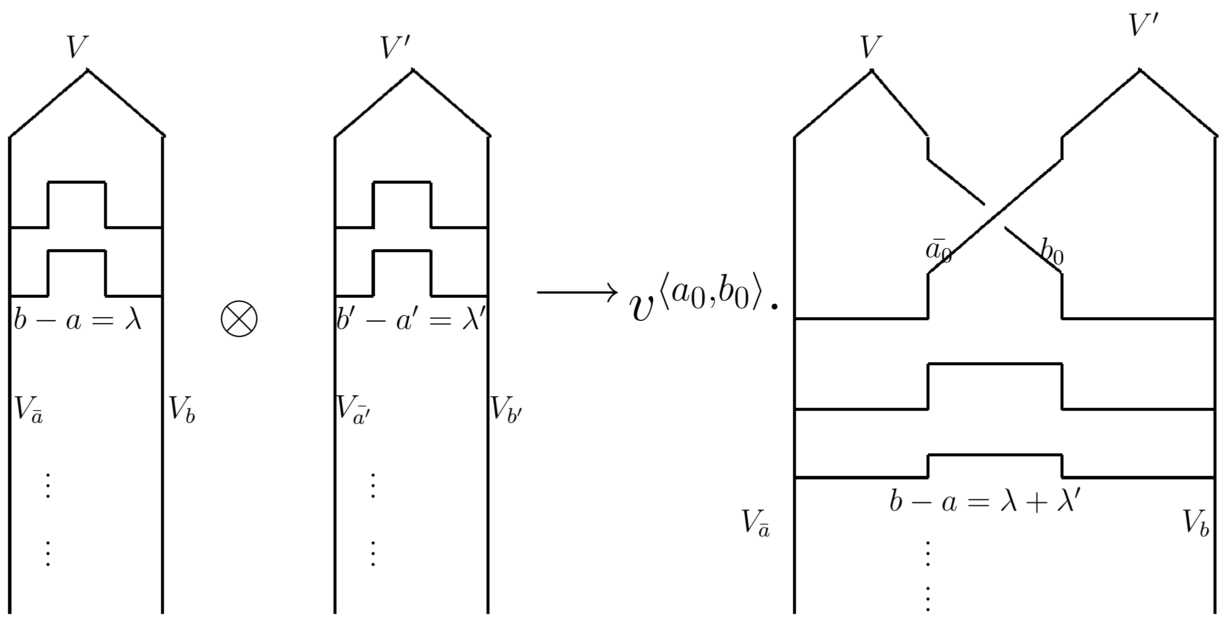


$V^{(a,b)} := Hom_{\mathscr{D}}(V_{\bar{a}} \otimes V_b, V)$, §25.1

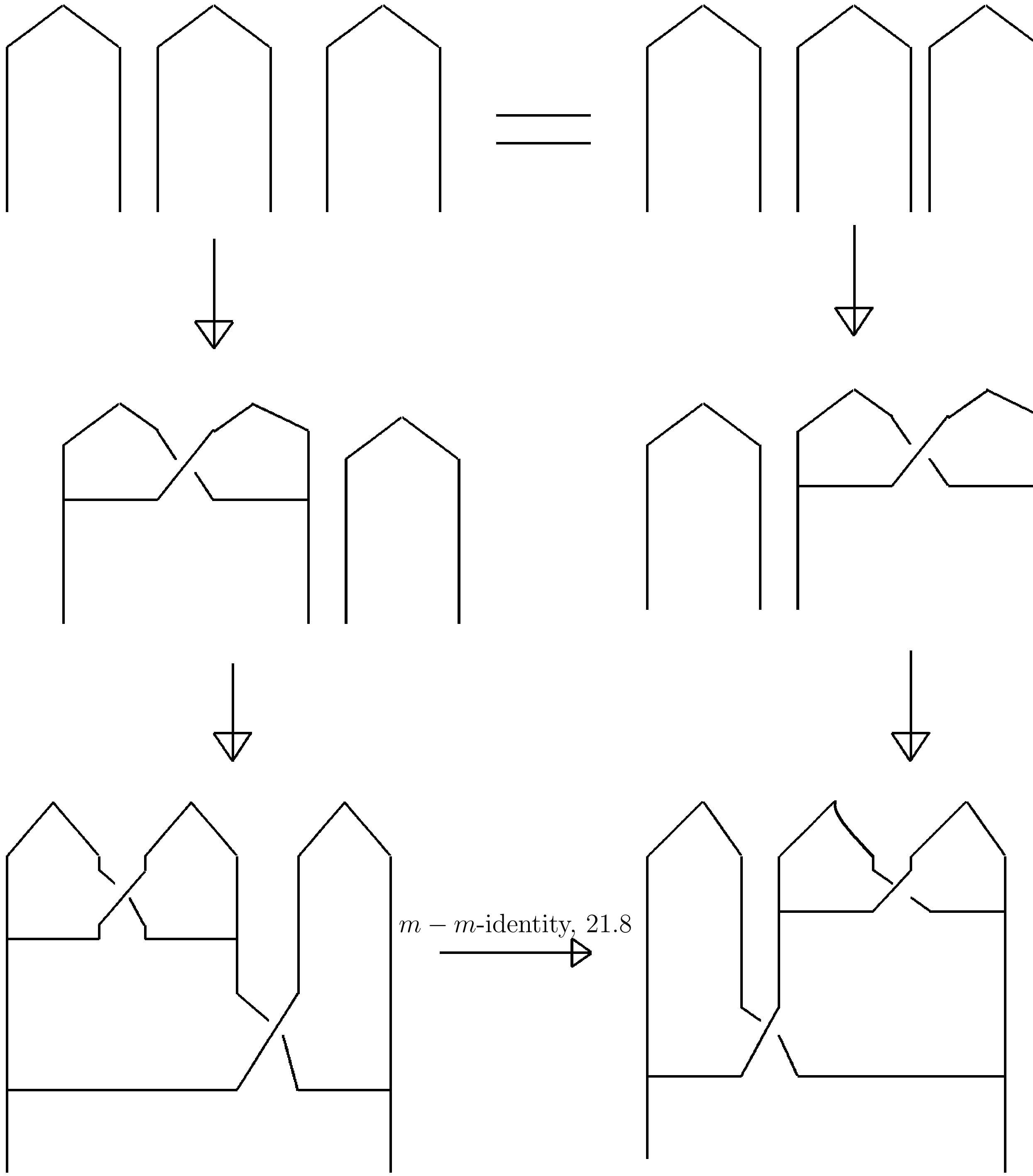




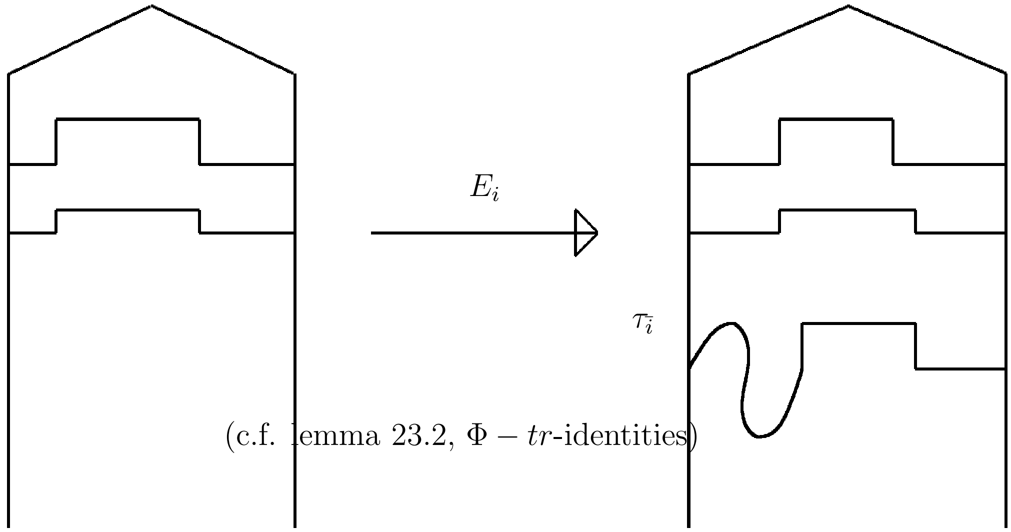
The tensor product $X_\lambda(V) \otimes X_{\lambda'}(V') \to X_{\lambda+\lambda'}(V \otimes V')$, 25.5(a)



Associativity of the functor X, Prop.25.8



The morphism E_i, representing a quantum group element



V_+ (Section 26, p.384) is defined as $Q_1^{\#}V$

V_0 (section 26) is the $\tilde{\mathfrak{g}}^+$-invariant subspace of V such that $V = V_+ + V_0$

$\Omega^{(2)}$ (26.3) is defined as $\sum_p c_p \otimes c_p \in U(\mathfrak{g}) \otimes U(\mathfrak{g})$

$\hat{L}_0$ (26.3) is defined as $\varprojlim L_0(N)$.

f_N (26.3) is defined as the natural map $\hat{W}(-\infty) \to W/G_N W$.

T_0 (26.5) is defined as the submodule of $\hat{L_O}$-finite vectors in $\hat{W}$.

$\mathscr{D}$ (26.4) the category of finite dimensional complex vector spaces

$\mathscr{G}$ (remark following 26.4) the functor $\mathscr{O}_\kappa \to \mathscr{D} : V \mapsto V/G_1 V$.

$\beta(V_1, V_2)$ (remark after 26.4) is the natural transformation (isomorphism) $\mathscr{G}(V_1) \otimes \mathscr{G}(V_2) \to \mathscr{G}(V_1 \dot{\otimes} V_2)$

V_i (26.4) is defined as Weyl modules $\mathbf{V}^{\kappa}_{a_i}$

$\bar{Y}$ (26.4) is defined as $\bigotimes_i \mathscr{V}_{a_i}$

Y (26.4) is defined as $\bigotimes_i \mathbf{V}^{\kappa}_{a_i}$

π (26.4) is defined as the natural projection $Y \to Y/\Delta Y$

$f_i j$ (26.4) a function on $\mathscr{V}$

$c_i(F)$ (26.4, p.385) is a linear operator: $\bar{Y} \to Y/\Delta Y : c_i(F)c(x' \otimes x_i \otimes x'') = \pi(x' \otimes (Fc)x_i \otimes x'')$ where $1 \leq i \leq n$ and $F \in A((\epsilon))$.

$\hat{\omega}$ (p.387) is defined as $-\frac{1}{\kappa}\left(\sum_{i=1}^{n} \Omega_i \omega_{i,0} + \sum_{i=1}^{n} f_{i,j} \Omega^{(2)}_{i,j} \omega_{i,-1}\right)$. Since it commutes with the action of $\mathfrak{g}$ on $\hat{Y}$, it defines a connection Δ on the bundle $A \otimes \bar{Y}/\mathfrak{g}\bar{Y} \cong Y/\Delta Y$. It coincides (cor. 26.1) with the connection defined in §12.11.

$\tilde{\nabla}$ (p.387) is defined as $\nabla_0 + \hat{\omega}$.

∇^{tr} (p.387) is the trivial connection on $A_\infty \otimes \bar{Y}/\mathfrak{g}\bar{Y}$

$\varphi(t)$ (387) is defined as $-\frac{q}{\kappa}\left(-\frac{t}{1-t}\Omega^{(2)}_{13} + \Omega_{34}\right)$. Its sum with the trivial connection ∇^{tr} is equal to (Lem.26.6) $\nabla_{t\partial/\partial t}$

φ_t (387) is defined as the map: $\bar{\mathscr{M}}(0) \to \mathscr{M}(0)$ defined in §15.25, p.984 (???), which can be considered as an endomorphism on $\bar{Y}/\mathfrak{g}\bar{Y}$.

$\mathscr{A}_\kappa$ (§27, p.388) is the subcategory of $\mathscr{O}_\kappa$ consisting of modules with Weyl filtrations

$\mathscr{A}^t_\kappa$ (391) is defined as $\mathscr{A}_\kappa \cap \mathscr{O}^t_\kappa$

$\mathscr{P}^\kappa_a(\lambda)$ (388) is defined as $U(\hat{\mathfrak{g}}^-) \otimes_{U(\mathfrak{g})} \mathscr{V}_a$

$\nabla(b), \Delta(b), \mathscr{F}(\Delta)$ (pp.390-391) are, respectively, A-modules and category defined in C.M.Ringel, *The category of modules with good filtrations ove a quasi-hereditary algebra has almostsplit sequences* , Math. Z. 208 (1991), pp. 209-223

$\tilde{\mathscr{N}}$ (392) the U^κ-module induced from the $U(\mathfrak{g})$-module $\mathscr{N}$.

S_a^κ (27.6, p.392) is defined as $Hom_{\mathscr{O}_\kappa}(\mathbf{V}_a^\kappa, D((V)_{\bar{a}}^\kappa))$. It can be considered as a bilinear form $s : \mathscr{V}_a^\kappa \times \mathscr{V}_{\bar{a}}^\kappa \to \mathbb{C}$ (p.393).

$\lambda_a(n)$ (393) is defined as $\frac{1}{2\kappa}\langle a, a+2\rangle + m$

$\check{s}_a^\kappa$ (393) is defined as the nonzero morphism $\mathbf{V}_a^\kappa \to D(\mathbf{V}_{\bar{a}}^\kappa)$ defined by s_a^κ (Lem.2.32 (c)).

$\mathbf{V}_{a,m}$ (393) is defined as ${}_{\lambda_a(m)}\mathbf{V}_a^\kappa$

$d_m(a)$ (393) is defined as $dim\mathbf{V}_{a,m}$

$D_{\alpha,m}(a)$ (393) is defined as $\sum_{c\in Y_\alpha(a)} d_m(c)$ when α is a dominant weight; it can be extended (p.393) to the whole weight lattice with the action of the Weyl group

W (393) the Weyl group for $\mathfrak{g}$

W_κ (393) a certain subgroup of the group of affine automorphisms of $\mathbb{Z}^I$, depending on the rationality of κ

Z (393) set of W_κ orbits on the weight lattice $\mathbb{Z}^I$.

$\mathscr{O}_\kappa(z)$ (393) a certain full subcategory of $\mathscr{O}_\kappa$ where all irreducible subquotients of V are isomorphic to L_a^κ for some $a \in z \cap \mathbb{N}^I$; where $z \in Z$ is a W_κ-orbit on $\mathbb{Z}^I$.

$\mathbb{N}_0$ (393) a subset of $\mathbb{N}^I$, depending on the rationality of κ

$C_{a,b,c}$ (393) is defined as $dimHom(\mathscr{V}_a \otimes \mathscr{V}_b \otimes \mathscr{V}_c, \mathbb{C})$

$\mathscr{O}_\kappa^a, \mathscr{O}_\kappa^{<a}$ (396) are full subcategories of $\mathscr{O}_\kappa$ with composition factors of some given form, their highest weights being restricted to some subsets

I_n (398) is defined as the set $\{k \in K | kn \in \mathbb{N}\}$

R (397) the ring of analytic functions over an unbounded open subset $S \subset \mathbb{C}$ meromorphic at infinity

R_κ (398) is defined as the completion of R at κ

$M(K)$ (398) is defined as $M \otimes_R K$

${}_{\Lambda}M^{\perp}$ (399) is defined as $\bigoplus_{\substack{\Lambda' \in T(A,M) \\ \Lambda \neq \Lambda'}} {}_{\Lambda'}M$.

$d^K_M(A, \Lambda)$ (399) is defined as $d_{M(K)}(A_K, \Lambda)$.

S (400) the open set $\mathbb{C} - \mathbb{R}_{\geq 0} \subset \mathbb{C}$

R (400) is defined as R_S = the ring of analytic functions on S, meromorphic at infinity (p.397)

$\hat{\mathfrak{g}}$ (400) is defined as $\hat{\mathfrak{g}}_R$

$\mathscr{O}$ (400) the full subcategory of $\tilde{\mathscr{O}}$ of $\hat{\mathfrak{g}}$ modules, consisting of those which are free as R-modules, and for any $\kappa \in S$, the module $V(\kappa)$ belongs to the category $\mathscr{O}_\kappa$.

φ (401) is some ring homomorphism $R \to \tilde{R}$

$\mathscr{O}$ (401) is the category of $\hat{\mathfrak{g}}_{\tilde{R}}$-modules defined analogously to $\mathscr{O}$

$\mathscr{A}_\varphi$ (401) a subcategory of $\mathscr{O}_\varphi$.

$\hat{\kappa}$ (401) the natural embedding $R \to R_\kappa$

$\tilde{\mathscr{N}}$ (401) the object in $\mathscr{O}_\varphi$ corresponding to the nilmodule $\mathscr{N}$.

$\mathscr{O}_\infty$ (401) is defined as $\mathscr{O}_{\kappa_\infty}$

$H(V, W)$ (401) is defined as $Hom_{\mathscr{O}}(V, W)$. It is (lemma 29.8) a free R-module of finite rank.

$r(\kappa)$ (401) is a natural embedding $H(V, W)(\kappa) \to Hom_{\mathscr{O}_\kappa}(V(\kappa), W(\kappa))$. (lem.29.8); tensoring with R_κ, we have an isomorphism $r(\hat{\kappa}) : H(V, W) \otimes_R R_\kappa \tilde{\to} Hom_{\mathscr{O}_{\hat{\kappa}}}(V \otimes_R R_\kappa, W \otimes_R R_\kappa)$.

$\mathscr{O}^a$ (402) the full subcategory of $\mathscr{O}$, consisting of modules V such that, for any $\kappa \in S$, $V(\kappa)$ lies in $\mathscr{O}^a_\kappa$. (§29.8, §28)

$M_i^{(n)}$ (403) is defined as $M_i/(x - \kappa)^n M_i$ where i is equal to 1 or 2.

$\alpha^{(n)}$ (403) is a morphism $M \to M_2^{(n)}$ of $\hat{\mathfrak{g}}_{R_\kappa}$-modules (prop.29.3)

$\hat{\gamma}$ (404) is defined as $x\hat{\beta}^{(n+1)}(m) - \hat{\beta}^{(n+1)}(xm)$

$\beta(\kappa)$ (404) is a morphism: $M(\kappa) \to M_1(\kappa)$ such that $\alpha(\kappa) = \varphi(\kappa) \circ \beta(\kappa)$

F_κ (404) field of fractions of R_κ

$\check{M}$ (404) image in $\mathscr{O}_{\check{\kappa}}$ of any object M in $\mathscr{O}_{\hat{\kappa}}$

Λ, F, M, A (405) Λ is a ring of formal power series over $\mathbb{C}$; F its field of fractions; and M a free finitely generated module over Λ with a filtration $0 = M_0 \subset M_1 \subset \cdots \subset M_N = M$ such that the quotients are torsion free; A an endomorphism of M preserving the filtration and such that for every $i : 1 \leq i \leq N$, the quotient modules $\bar{M}_i = M_i/M_{i-1}$ decomposes as $\bar{M}_i = \bigoplus_{j=1}^{n_i} M^{\alpha^i_j}$

Q (405) is defined as the set of pairs (i,j) such that $1 \leq i \leq N$, $1 \leq j \leq n_i$, and $(i,j) \neq (1,1)$.

$m^Z_{(i,j)}$ (405), where Z is a subset of Q, is an endomorphism of $\check{M}$, defined as follows: $\alpha^i_j - \check{B}$ for $(i,j) \notin Z$; and $-\check{C}$ for $(i,j) \in Z$.

B (405)

Q_0 (405)

N_j (405)

$\tilde{i}$ (406) is a natural embedding: $\mathbf{1} \to M$ (corollary for lemma 29.12); We have $e \circ \tilde{i} = (x-\kappa)^{N-1}$.

e (405) is the natural projection $e : M = M_N \to M_N/M_{N-1} = \mathbf{1}$; We have $e \circ \tilde{i} = (x-\kappa)^{N-1}$.

$\mathbf{V}^{\hat{\kappa}}_a$ (406) is defined as $\mathbf{V}_a \otimes_R R_\kappa$ (remark after corollary to lemma 29.12)

$\hat{s}^{\hat{\kappa}}_a$ (406) is defined as $\hat{s}_a \otimes 1$

$\check{s}^{\hat{\kappa}}_a$ (406) is an isomorphism: $\mathbf{V}^{\hat{\kappa}}_{\bar{a}} \to D(\mathbf{V}^{\hat{\kappa}}_{\bar{a}})$ defined by $\hat{s}^{\hat{\kappa}}_a$ when $\mathbf{V}^{\hat{\kappa}}_a$ is irreducible.

$\mathscr{F}$ (406, §30) is defined as the functor: $V \mapsto V(1)$ from $\mathscr{O}_\kappa$ to $\mathscr{D}$, the category of finite dimensional representations of the Lie algebra $\mathfrak{g}_F := \mathfrak{g} \otimes_{\mathbb{C}} F$ where κ is a ring morphism: $R \to F$ from ring R to a field F.

$\mathscr{G}$ (406, §30) is defined as the functor: $V \mapsto V/Q_1^{\#}V$ from $\mathscr{O}_\kappa$ to $\mathscr{D}$, the category of finite dimensional representations of the Lie algebra $\mathfrak{g}_F := \mathfrak{g} \otimes_{\mathbb{C}} F$ where κ is a ring morphism: $R \to F$ from ring R to a field F. F and G are equivalences of categories; α is a functorial isomorphism between them.

α (406, §30) is defined as $q_V \circ p_V$ where $V \in \mathscr{O}_\kappa$, $p_V : \mathscr{F}(V) \to V$ and $q_V : V \to \mathscr{G}(V)$

θ_1, θ_2 (407) are two functors from $\mathscr{O}_\kappa \times \mathscr{O}_\kappa$ to $\mathscr{D}$, taking, respectively, a pair of objects to the tensor product of its target by the functor $\mathscr{G}$ and to the target of the fusion tensor product of the pair.

$\beta(V_1, V_2)$ (407) is an isomorphism $\mathfrak{g}(V_1) \otimes \mathfrak{g}(V_2) \to \mathfrak{g}(V_2 \dot{\otimes} V_2)$ where V_1, V_2 are Weyl modules; It defines an isomorphism between functors θ_1, θ_2.

$\tilde{\beta}$ (407) is an isomorphism: $\tilde{W}_1 \dot{\otimes} \tilde{W}_2 \tilde{\to} (W_1 \tilde{\otimes} W_2)^\kappa$, where $W_1, W_2 \in \mathscr{D}$

R_∞ (407, §31) completion of R at infinity

$\mathscr{P}$ (407, §31) commutativity isomorphism for $(V_1 \otimes V_2)$

φ (407, §31) isomorphism $\left\langle V_1 \dot{\otimes} V_2, V_3 \dot{\otimes} V_4 \right\rangle \tilde{\to} \langle V_1, V_2, V_3, V_4 \rangle$ defined as in §17.29 (p.1008), for objects $V's$ in $\mathscr{A}$.

a (407, §31) associativity isomorphism for $(V_1 \dot{\otimes} V_2) \dot{\otimes} V_3)$, defined as in §18.2, for objects $V's$ in $\mathscr{A}$.

$\hat{P}$ (407, §30) commutativity isomorphism for fusion tensor products of objects in $\mathscr{O}_\infty$.

$\hat{a}$ (407, §30) associativity isomorphism for fusion tensor products of objects in $\mathscr{O}_\infty$.

A_{an} (407, §30) ring of R_∞-valued analytic germs at $0 \in \mathbb{C}$

M (407) finitely generated torsion-free R_∞-module

M_{an} (407) is defined as $M \otimes_{R_\infty} A_{an}$

$\nabla^0_{t\partial/\partial t}$ denotes the trivial connection on M_{an}

$\nabla_{t\partial/\partial t}$ (407) a connection on $\mathscr{M}_{an}$

$\nabla'_{t\partial/\partial t}$ (407) a connection on $\mathscr{M}_{an}$

$(\mathscr{N}, \nabla_{\partial/\partial t})$ (408) an R-vector bundle over $\mathbb{C} - \langle 0, 1\rangle$ with connection $\nabla_{\partial/\partial t}$ being (the trivial connection) + $-\frac{1}{\kappa}(\frac{\Omega_{12}}{t} - \frac{\Omega_{23}}{1-t})dt$

$\langle \mathscr{N} \rangle$ (408) space of flat sections $n(t)$ of $\tilde{\mathscr{N}}$ over the interval $(0, 1)$.

a_0, a_1 (409) linear isomorphisms from $\langle \mathscr{N} \rangle \to \mathscr{N}$ given by the asymtotics

$$a_0(n(t)) = \lim_{t \to 0}(t^{\Omega_{12}/\kappa} n(t));$$

$$a_1(n(t)) = \lim_{t \to 1}((1-t)^{\Omega_{23}/\kappa} n(t)).$$

It turns out that (corollary, p.409) $a^{\infty} = a_1 \circ a_0^{-1}$.

$(\mathscr{O}_{\infty}, \dot{\otimes}, a, \mathscr{P})$ (410) is a braided category (corollary, p.410); $(\mathscr{G}, \beta)$ defines an equivalence between $\mathscr{O}_{\infty}$ and $\mathscr{D}$ (corollary, p.410); where $\beta : \mathscr{G}(\tilde{W}_1) \otimes \mathscr{G}(\tilde{W}_2) \to \mathscr{G}(\tilde{W}_1 \dot{\otimes} \tilde{W}_2)$ (lemma 30.2, p.407)

κ_0 (410) is an element in $\mathbb{C} - \mathbb{Q}_{\geq 0}$

φ_a (411) is defined as the composition ... ,which is a map V_a^{∞} to itself. It is part of the "rigidity" structure.

δ_a (411) is defined as $\varphi_a = \delta_a Id$, where $\delta_a \in R_{\infty}$. Its order of pole at κ plays a key role in the numerical criterion of rigidity in Prop. 31.3, p.412.

ρ (411) is the sum of fundamental weights

$\lambda_{2\rho}$ (411) is defined as $\prod_{i \in I} \lambda_i^{c_\imath}$

τ (411) is defined as $\lambda_{2\rho}(v)$, an R_{∞}-point in G; where $v = e^{-\pi i/\kappa}$

d_a (412) is defined as δ_a^{-1}. It turns out (31.6, p.413) that $d_a(\kappa) = \frac{v^n - v^{-n}}{v - v^{-1}}$

$n_a(\kappa)$ (412) the order of zero of d_a at κ

$\hat{i}$ (412)

$\check{s}_a^{\hat{\kappa}}$ (412) is an isomorphism $\mathbf{V}_{\hat{a}}^{\hat{\kappa}} \xrightarrow{\sim} D(\mathbf{V}_a^{\hat{\kappa}})$.

$i_{\mathbf{V}_a}$ (412) is a morphism $\mathbf{1} \longrightarrow \mathbf{V}_a^{\hat{\kappa}} \dot{\otimes} D(\mathbf{V}_a^{\hat{\kappa}})$.

$e_{\mathbf{V}_a}$ (412) is a morphism $\mathbf{V}_a^{\hat{\kappa}} \dot{\otimes} D(\mathbf{V}_a^{\hat{\kappa}}) \longrightarrow \mathbf{1}$.

P (412) is the root lattice

$\tilde{W}$ (412) is defined as the semidirect product $W \ltimes P$, the affine Weyl group

W_κ (412) a subgroup of the affine Weyl group, depending on the rationality of κ

Ω_κ (412) the intersection of the W_κ-orbit of 0 with with $\mathbb{N}^I$.

$\bar{V}_i$ (414) quotient of two successive objects V_i in a composition series

j (415) is defined as $e \otimes 1 \otimes e \circ 1 \otimes i \otimes 1$,a morphism from $(\mathbf{V}_a^\kappa)^* \dot{\otimes} \mathbf{V}_a^\kappa \longrightarrow T^\kappa \otimes T^{\kappa *}$

$\mathbb{N}^I(\theta)$ is defined as $\left\{a \in \mathbb{N}^I | \ \theta_a|_{Z_\theta} \equiv 1\right\}$

L (415) is defined as $ker(e_{\mathbf{V}_a^\kappa}) \subset (\mathbf{V}_a^\kappa)^* \otimes \mathbf{V}_a^\kappa$

T (415) lifting (prop.29.2) of T^κ to category $\mathscr{O}_{\hat{\kappa}}$

$\check{i}$ (415) is a morphism: $\mathbf{1} \to \mathbf{V}_a^{\hat{\kappa}} \dot{\otimes} T \dot{\otimes} T^* \dot{\otimes} (\mathbf{V}_a^{\hat{\kappa}})^*$,an extension of i.

$\hat{j}$ (415) is a morphism: $(\mathbf{V}_a^{\hat{\kappa}})^* \otimes \mathbf{V}_a^{\hat{\kappa}} \to T \otimes T^*$

$\hat{L}$ (415) is defined as $ker(e_{\mathbf{V}_a^{\hat{\kappa}}}) \subset (\mathbf{V}_a^{\hat{\kappa}})^* \otimes \mathbf{V}_a^{\hat{\kappa}}$

$\tilde{j}$ (415) is defined as $\hat{j} \otimes F_\kappa$

$\tilde{L}$ (415) is defined as $\hat{L} \otimes F_\kappa \subset ker(\tilde{j})$

θ (415) is defined as θ_c which is the character of $Z \in G$ (§31, p.411): $\rho_c(z) = \theta_c(z) \cdot Id$

$\mathbf{V}^N$ is defined as $(V_c^\kappa)^{\otimes N}$

$\mathscr{O}_\kappa^\theta$ is a subcategory of $\mathscr{O}_\kappa$, consisting of modules whose irreducible subquotients are isomorphic to L_a where $a \in \mathbb{N}^I(\theta)$

P^κ (416) projective module covering the irreducible quotient $\mathbf{L}_b^\kappa$.

φ^- (416) an involution of D

Δ (416) discriminant function on T

$\tilde{\mathbf{N}}$ (417) is defined as $\{a \in \mathbf{N}^I(\theta) | T_a^\kappa$ is projective$\}$

$\bar{\mathbf{N}}$ (417) is defined as the image of $\tilde{\mathbf{N}}$ in $\mathbf{N}^I(\theta)/p\mathbf{N}^I(\theta)$

$\mathbf{N}^I(m)$ (418) a subset

m_0 (418) a constant

$\mathscr{T}_{a,b}$ (420) is defined as $Hom_{\mathscr{O}}(\mathbf{V}_{a+b}, \mathbf{V}_a \dot{\otimes} \mathbf{V}_b)$

$\mathscr{S}_a$ (420) is defined as $Hom_{\mathscr{O}}(V_{\bar{a}}, D(V_a)) = Hom_{\mathscr{O}}(V_{\bar{a}} \dot{\otimes} V_a, \mathbf{1})$

$\check{T}$ (420) a generator of $\mathscr{T}_{a,b}$

$t_{a,b}$ (421) a fixed element in $\mathscr{T}_{a,b}$

φ (421) an embedding $\mathscr{V}_{a+b} \to \mathscr{V}_a \otimes \mathscr{V}_b$.

$\check{s}'_a$ (421) a generator of the free R-module $\mathscr{S}_a$; its choice is fixed in corollary to lemma 33.5.

r (421) a positive rational number, depending on the type of the root system

Z_r (421) a right-infinite subset of the real line

$\mathbf{V}_a$ (421)

$\check{V}$ (421)

$(\check{\mathscr{G}}, \check{\beta})$ (421) a morphism from the braided category $\check{\mathscr{O}}$ to the Drinfeld category

$\mathscr{D}_R$ (421) category of finitely generated R-modules

$\check{s}_a$ (422) choice of $\check{s}'_a$, such that all $g_{a,b} \equiv 1$.

$\theta_{i;a,b}$ (423) is defined as $Hom_{\check{\mathscr{O}}}(V_{a+b-\alpha_i}, V_a \otimes V_b) \otimes \check{R}$

$\mathscr{G}(\theta)$ (423) an element in $Hom(V_{a+b-\alpha_i}, V_a \otimes V_b) \otimes \check{R}$, the image of $\theta \in \theta_{i;a,b}$.

$\tau_{i;a,b}$ an element in $Hom(V_{a+b-\alpha_i}, V_a \otimes V_b) \otimes \check{R}$, defined as a quotient of τ' by Gamma functions

$\check{\tau}_{i;a,b}$ (423) an element in $\theta_{i;a,b}$, mapping to τ (p.423), it exists by prop. 33.1

η (423) a morphism: $\mathbf{V}_{a+b-\alpha_i} \to W$

$F_{a,b}(\kappa)$ (424) is a rational function of Gamma factors

$V \sim_a 0$ (424) a-equivalence; V is a-equivalent to zero if $Hom_{\mathscr{O}_{\kappa_0}}(V^{\kappa_0}_{\bar{a}-\bar{\alpha}_i}, V) = \langle 0 \rangle$.

a-isomorphism (424) a morphism between objects in $\mathscr{O}_{\kappa_0}$ whose ker and coker are both equivalent to zero

s_a (425) a choice of generators in $\mathscr{S}_a$

$\check{tr}^c_{a,b}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

$\check{m}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

$\check{\varphi}^e_{i;a,b}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

$\check{\psi}^e_{i;a,b}$ (425) morphism defined with the generalised T, S, τ morphisms, in the same way as the "unchecked" morphism in Part III.

$V^{(a,b)}$ (425) is defined as $Hom_{\check{\mathscr{O}}}(V_{\bar{a}} \dot{\otimes} V_b, V)$

$\mathbf{X}_\lambda(V)$ (426) is defined as $\varinjlim_{b-a=\lambda} V^{(a,b)}$

$\mathbf{X}(V)$ (426) is defined as $\bigoplus_{\lambda \in \mathbb{Z}^I} \mathbf{X}_\lambda(V)$

$M_{V,V'}$ (426)

$\check{\Phi}^c_{i;a,b}$ (426)

E^V_i (426)

$\check{\Psi}^c_{i;a,b}$ (427)

F^V_i (427)

$\check{v}_i$ (427) is an automorphism of the functor $\mathbf{X}$

$W^{(a,b)}$ (428) is defined as the complex vector space $Hom_{\mathscr{O}_\kappa}(V_{\bar{a}}^{\kappa}\dot{\otimes}V_b^{\kappa}, W)$.

$\bar{\mathbf{X}}_\lambda(W)$ (428) is defined as $\varinjlim_{b-a=\lambda} W^{(a,b)}$

$\bar{\mathbf{X}}(W)$ (428) is defined as $\bigoplus_{\lambda\in\mathbb{Z}^I} \bar{\mathbf{X}}_\lambda(W)$

$\bar{E}_i^W$ (428) defined analogously to E_i^V

$\bar{F}_i^W$ (428) defined analogously to F_i^V

$\bar{M}_{W,W'}$ (428) the unique functorial isomorphism $\bar{X}(W)\otimes\bar{X}(W') \to \bar{X}(W\dot{\otimes}W')$ in $\mathscr{O}_\kappa$, satisfying a commutation relation between the M and r morphisms. (429)

$\mathscr{V}_c^\lambda$ (429) is defined as $\{v \in \mathscr{V}_c | h_i v = \lambda(i)v; i \in I\}$

$C_{\bar{a},b,\bar{c}}$ (429) is defined as $dim_{\mathbb{C}}(\mathscr{V}_c^\lambda)$

$s_a(\kappa)$ (429)

φ_c (429)

x_c^0 (429) is an element in $(V_{\bar{c}}^{\kappa})^{(c,0)}$

y_c^0 (429) is an element in $(V_{\bar{c}}^{\kappa})^{(0,c)}$

κ (430) is a complex number

$\bar{\mathscr{E}}(\kappa)$ (430) is a category of graded finite-dimensional vector spaces

$d(a,\kappa)$ (430) is defined as $dim L_a$

$\mathbf{X}_c$ (431) is defined as $\bar{\mathbf{X}}_c(V_c^\kappa)$

$\mathbf{X}_c^*$ (431) is defined as the object $\bar{\mathbf{X}}_c((V_c^\kappa)^*)$ in $\bar{\mathscr{E}}_\kappa$

φ (431) is a pairing between $\mathbf{X}_c$ and $\mathbf{X}_c^*$ defined via the morphisms M and $\check{s}_c$.

stable object (432) a V in $\mathscr{O}_\kappa$ such that for every V', the map $M_{V,V'}$ of $\bar{\mathbf{X}}(\_)$'s is an isomorphism

ψ (432)

$\hat{\psi}$ (432)

$\mathbf{X}(V)(\kappa)$ (432) is defined as $\mathbf{X}(V)/m_\kappa \mathbf{X}(V)$

P (433) is a polynomial in x,y, with coefficients in the ring of rational functions in $\mathbb{C}$.

A (433) is defined as $\mathbb{C}[v, v^{-1}]$

$[n]$ (433) is defined as $\frac{v^n - v^{-n}}{v - v^{-1}}$ for an integer n

$\mathbf{f}$ (433) is an algebra over $\mathbb{C}(v)$ with generators $\kappa_i : i \in I$ and relations

$$\sum_{p,p' \in \mathbb{N}; p+p'=1-a_{ij}} (-1)^{p'} \frac{\kappa_i^p}{[p]!} \kappa_j \frac{\kappa_i^{p'}}{[p']!} = 0$$

${}_A\mathbf{f}$ (434) is the A-subalgebra of $\mathbf{f}$ generated by the divided powers $\kappa_i^{(p)} = \frac{\kappa_i^p}{[p]!}$

$\mathbf{f}_\nu$ (434) ???

${}_R\mathbf{f}$ (434) is an R-algebra defined by a change of scalar; where R is a commutative algebra over A

${}_R\mathscr{C}$ (434) is an abelian category of finitely generated R-modules with action of ${}_R\mathbf{f}$.

$(,)$ (435) is a bilinear form on $\mathbf{f}$

${}_iR$ (435) is a $\mathbb{C}(v)$-linear map of $\mathbf{f}$

$(b), (b^*)$ (435) dual bases of $\mathbf{f}_\nu$

T_ν (435) is an element in ${}_A\mathbf{f}_\nu \otimes_A ({}_A\mathbf{f}_\nu)$, defined with the dual bases b, b^* of $\mathbf{f}_\nu$.

ϕ (435) is a bilinear pairing of X with values in $\frac{1}{c}\mathbb{Z}$

Pi_ϕ (435) is defined as $(m \otimes m') \mapsto v^{\phi(\lambda,\lambda')} m \otimes m'$ where the lambda's are weights of the elements m's

$\mathbf{s}$ (435) is the switching map for a tensor product of two modules

Θ (435) is a morphism for the tensor product of two modules, defined with the maps T_ν

$\mathscr{R}_{M',M}$ (435) is defined as $\Theta \circ \Pi_\phi \circ \mathbf{s}$

$E_i^{(p)}$ (436) as an operator $(M^*)^\lambda \to (M^*)^{\lambda+pi'}$, is defined as:

$$(E_i^{(p)} m^*)(m) = (-1)^p v^{p\lambda(i)+p(p-1)} m^*(E_i^{(p)} m)$$

for $m^* : M^{-\lambda} \to R, m \in M^{-\lambda+pi'}$

$F_i^{(p)}$ (436) is defined analogously to $E_i^{(p)}$

i_M (436) is a morphism $\mathbf{1} \to M \otimes_R M^*$

$\mathscr{C}_\kappa$ (436) is a braided category (of modules over a quantum group)

ζ (436) is defined as $exp(-i\pi/\delta\kappa)$

$\mathscr{L}$ (436) is an irreducible object in $\mathscr{C}_\kappa$

$\tilde{\mathbf{X}}$ (437) is the functor

$$W \longrightarrow \tilde{\mathbf{X}} = \left(\bar{\mathbf{X}}(W) = \bigoplus_\lambda \bar{\mathbf{X}}_\lambda(W), (\bar{E}_i^W)^{(n)}, (\bar{F}_i^W)^{(n)} \right)$$

(see corollary to prop. 36.1). The pair $\tilde{\mathbf{X}} = (\bar{\mathbf{X}}, \bar{M})$ is a braided functor: $\mathscr{O}_\kappa \to \mathscr{C}_\kappa$ (lemma 38.1, p.437). It turns out to be an equivalence of categories (Theorem 38.1, p.438).

P_γ^κ (438)

Q_γ^κ (438)

c_i (438)