AMS261 Practice Midterm 2 v2.

This is a practice exam for Midterm 2. The actual exam will be very similar to this exam.

You may use a calculator for this exam. You must show work to receive full credit. An answer without any kind of work will not be credited, even if it is correct.

1. Let $f(x, y) = x^2 + y^2$ and $R = \{x^2 + y \geq 1\}$.
   (a) Does f necessarily have an absolute maximum or an absolute minimum on R? Explain.
   (b) Find the critical points f inside the region R.
   (c) Find the absolute max/min of f on region R.
   (d) Let $R' = \{x^2 + y = 1.0001\}$. Estimate the absolute maximum and/or the minimum of f on R'.

2. Consider the integral

$$\int_0^5 \int_0^{4\sqrt{1-z^2/25}} \int_{-5\sqrt{1-y^2/16-z^2/25}}^{5\sqrt{1-y^2/16-z^2/25}} \frac{1}{\sqrt{\frac{x^2}{25} + \frac{y^2}{16}}} dx\ dy\ dz$$

   (a) Sketch the region on which the integration is being performed.
   (b) Use appropriate coordinate transformation to rewrite the integral and evaluate it.

3. One possible parameterization of the cylinder with an ellipse base $x^2 + \dfrac{z^2}{25} = 1$ is:

$$\vec{r}(s, t) = \sin(t)\hat{i} - s\hat{j} + 5\cos(t)\hat{k}$$

   (a) Show that the points of $\vec{r}(s, t)$ actually lie on $x^2 + \dfrac{z^2}{25} = 1$.
   (b) Find a parameterization of the tangent plane at the point $(1, -1, 0)$.
   (c) Set $s = -1$. This gives you a curve.
      i. Find the equation of the tangent line of this curve at $(1, -1, 0)$.
      ii. Find the length of the curve from the point $(1, -1, 0)$ to the point $(1, 3, 0)$.

4. A particle is moving along a square of length 1 in the first quadrant in the xy-plane in the counter-clockwise direction. The particle is subject to a force field of $\vec{F}(x, y) = y\hat{i} - 2x\hat{j}$

   (a) Sketch the curve and the vector field.

   (b) Is $\vec{F}$ a conservative force field? If so, find the potential function. If not, explain why.

   (c) Find the work done by the force on the particle starting from the point $(0, 0)$ and ending at point $(1, 1)$.

   (d) Find the work done by the force on the particle going around the square exactly once.