AMS261 Practice Midterm 2 ANSWERS

1. Let $f(x, y) = 4y^3 + 12y^2 + x^2$ and $R = \{\frac{x^2}{4} + y^2 \leq 1\}$.

   (a) Does f necessarily have an absolute maximum or an absolute minimum on R? Explain. (Hint: Is f continuous on R? Is R closed and bounded?)
   f is contiuous, and R is a closed and bounded region. So by the Extreme Value Theorem, f attains a maximum and a minimum on R.

   (b) Find the critical points f. Which of the critical points are inside of the region R?
   (0,0)

   (c) Let g denote the constraint function. What is $g(x, y)$ in this problem?
   $g(x, y) = \frac{x^2}{4} + y^2$

   (d) Find the local max/min on the boundary of region R.
   $(2, 0), (-2, 0), (0, 1), (0, -1)$

   (e) Find the absolute max/min of f on region R.
   $f(0, 0) = 0,\ f(0, 1) = 16$

   (f) Let $R' = \{\frac{x^2}{4} + y^2 \leq 1.00001\}$.

      i. Should absolute max/min of f on R be different from the one on R'?
      Absolute max changes. Absolute min shouldn't change.

      ii. Estimate the absolute max/min of f on R'.
      Abs max ≈ 16.00018
      Abs min $= 0$

2. Consider the integral

$$\int_0^5 \int_{-z}^{z} \int_{-3\sqrt{z^2-y^2}}^{3\sqrt{z^2-y^2}} dxdydz$$

(a) Sketch the region on which the integration is being performed.
It's a cone with a base in the shape of an ellipse. The ratio of the axes are 3:1, for x and y.

(b) Evaluate the integral. (Hint: You may have to use at least one coordinate change.)
125π

3. One possible parameterization of the sphere $x^2 + y^2 + z^2 = 25$ is:

$$\vec{r}(s,t) = \sqrt{25 - s^2}\sin(2\pi t)\hat{\imath} + \sqrt{25 - s^2}\cos(2\pi t)\hat{\jmath} + s\hat{k}$$

(a) Verify that $\vec{r}(s,t)$ is actually a parametrization of the sphere.

i. Show that the points of $\vec{r}(s,t)$ actually lie on $x^2+y^2+z^2 = 25$.

$$\begin{aligned}(\sqrt{25 - s^2}\sin(2\pi t))^2 + (\sqrt{25 - s^2}\cos(2\pi))^2 + s^2 &= (25 - s^2)\sin(2\pi t)^2 + (25 - s^2)\cos(2 && (1)\\ &= (25 - s^2)(1) + s^2 && (2)\\ &= 25 && (3)\end{aligned}$$

ii. Explain why every point of $x^2 + y^2 + z^2 = 25$ can be reached by $\vec{r}(s,t)$. (Hint: Can you figure out which coordinate system this parametrization is based on?)
This parametrization comes from restricting the cylindrical coordinates onto the sphere.

(b) The Tangent Plane at $(3, 0, 4)$.

i. Find the values of s and t so that $\vec{r}(s,t) = 3\hat{\imath} + 0\hat{\jmath} + 4\hat{k}$.
$s = 4$, $t = 1/4 + n$.

ii. Fix t to be the value found in the previous part on $\vec{r}(s,t)$. Find the tangent vector of this curve at the value of s found

in the previous part.
$\vec{v} = (-4/3, 0, 1)$

iii. Fix s to be the value found in part (i) on $\vec{r}(s,t)$. Find the tangent vector of this curve at the value of t found in part (i).
$\vec{w} = (0, -6\pi, 0)$

iv. Find a parameterization of the tangent plane at the point $(3, 0, 4)$.
$\vec{p}(s,t) = (3, 0, 4) + s(-4/3, 0, 1) + t(0, -6\pi, 0)$

(c) Set $s = 4$. This gives you a curve.

i. Find the equation of the tangent line of this curve at $(3, 0, 4)$.
$\vec{l}(t) = (3, 0, 4) + (0, -6\pi, 0)t$

ii. Find the length of the curve from the point $(0, 3, 4)$ to the point $(3, 0, 4)$
$9\pi/2$

4. A particle is moving long an ellipse parametrized by $\vec{r}(t) = 2\cos(t)\hat{\imath} + 4\sin(t)\hat{\jmath}$ through a force field $\vec{F}(x, y) = x\hat{\imath} - y\hat{\jmath}$

(a) Sketch the curve and the vector field.

(b) Find the curl of F.
0

(c) Is $\vec{F}$ a conservative force field? If so, find the potential function. If not, explain why.
Yes. $f(x, y) = (x^2 - y^2)/2 + C$.

(d) Find the work done by the force on the particle starting from the point $(2, 0)$ and ending at point $(-2, 0)$.
0

(e) Find the work done by the force on the particle going around the ellipse exactly once.
0