AMS261 Practice Midterm 2

This is a practice exam for Midterm 2. The actual exam will be very similar to this exam.

You'll notice that there are many parts to each problems. These parts are intended as a guide to solving the actual problem at hand, and also to give you an idea as to how I'll be grading the midterm. A lot of the parts will not appear on the actual exam.

You may use a calculator for this exam. You must show work to receive full credit. An answer without any kind of work will not be credited, even if it is correct.

1. Let $f(x, y) = 4y^3 + 12y^2 + x^2$ and $R = \{\frac{x^2}{4} + y^2 \leq 1\}$.
   (a) Does f necessarily have an absolute maximum or an absolute minimum on R? Explain. (Hint: Is f continuous on R? Is R closed and bounded?)
   (b) Find the critical points f. Which of the critical points are inside of the region R?
   (c) Let g denote the constraint function. What is $g(x, y)$ in this problem?
   (d) Find the local max/min on the boundary of region R.
   (e) Find the absolute max/min of f on region R.
   (f) Let $R' = \{\frac{x^2}{4} + y^2 = 1.00001\}$.
      i. Should absolute max/min of f on R be different from the one on R'?
      ii. Estimate the absolute max/min of f on R'.

2. Consider the integral

$$\int_0^5 \int_{-z}^{z} \int_{-3\sqrt{z^2-y^2}}^{3\sqrt{z^2-y^2}} dxdydz$$

   (a) Sketch the region on which the integration is being performed.
   (b) Evaluate the integral. (Hint: You may have to use at least one coordinate change.)

3. One possible parameterization of the sphere $x^2 + y^2 + z^2 = 25$ is:

$$\vec{r}(s,t) = \sqrt{25 - s^2}\sin(2\pi t)\hat{\imath} + \sqrt{25 - s^2}\cos(2\pi t)\hat{\jmath} + s\hat{k}$$

   (a) Verify that $\vec{r}(s,t)$ is actually a parametrization of the sphere.
      i. Show that the points of $\vec{r}(s,t)$ actually lie on $x^2+y^2+z^2 = 25$.
      ii. Explain why every point of $x^2 + y^2 + z^2 = 25$ can be reached by $\vec{r}(s,t)$. (Hint: Can you figure out which coordinate system this parametrization is based on?)
   (b) The Tangent Plane at $(3, 0, 4)$.
      i. Find the values of s and t so that $\vec{r}(s,t) = 3\hat{\imath} + 0\hat{\jmath} + 4\hat{k}$.
      ii. Fix t to be the value found in the previous part on $\vec{r}(s,t)$. Find the tangent vector of this curve at the value of s found in the previous part.
      iii. Fix s to be the value found in part (i) on $\vec{r}(s,t)$. Find the tangent vector of this curve at the value of t found in part (i).
      iv. Find a parameterization of the tangent plane at the point $(3, 0, 4)$.
   (c) Set $s = 4$. This gives you a curve.
      i. Find the equation of the tangent line of this curve at $(3, 0, 4)$.
      ii. Find the length of the curve from the point $(0, 0, -5)$ to the point $(3, 0, 4)$

4. A particle is moving long an ellipse parametrized by $\vec{r}(t) = 2\cos(t)\hat{\imath} + 4\sin(t)\hat{\jmath}$ through a force field $\vec{F}(x,y) = x\hat{\imath} - y\hat{\jmath}$

   (a) Sketch the curve and the vector field.
   (b) Find the curl of F.
   (c) Is $\vec{F}$ a conservative force field? If so, find the potential function. If not, explain why.
   (d) Find the work done by the force on the particle starting from the point $(2, 0)$ and ending at point $(-2, 0)$.
   (e) Find the work done by the force on the particle going around the ellipse exactly once.