AMS261 Practice Midterm 1

1. Let A and B be planes represented by equations $z = 2x - 2y + 5$ and $10 = -x + y - 4z$.

   (a) Find a normal vector to plane A.
   $< 2, -2, -1 >$

   (b) Find a normal vector to plane B.
   $< -1, 1, -4 >$

   (c) Find the dot product between the normal vectors.
   0

   (d) Find the cross product between the normal vectors.
   $< 9, 9, 0 >$

   (e) What is the angle between plane A and plane B?
   90 degrees, or $\frac{\pi}{2}$

   (f) Find a vector that is parallel to the intersection of planes A and B.
   $< 9, 9, 0 >$

   (g) Find a plane perpendicular to plane A and plane B.
   $9x + 9y = 0$

2. In class, we learned how to take the projection of one vector onto another vector. In this problem, you'll take the projection of one vector onto a plane. Let C be a plane given by the equation $50 = 2x - 4y + z$, and let $\vec{v} = 3\hat{i} + 2\hat{j} - 6\hat{k}$. Find the projection of $\vec{v}$ onto plane A.

   (a) Find a normal vector to the plane A
   $< 2, -4, 1 >$

   (b) Find the projection of vector v onto the normal vector found in the previous part.
   $\frac{-8}{21} < 2, -4, 1 >$

   (c) Find the projection of vector v onto the plane A. (Hint: Drawing a picture might be useful.)
   $< \frac{79}{21}, \frac{10}{21}, \frac{-118}{21} >$

3. There is a hiker climbing the surface $f(x, y) = 4 - x^2 + y$ at the point (1,2)

(a) The hiker wishes to climb the steepest ascent. What direction should the hiker move? (Note: Direction should be given as a unit vector.)

$\frac{(-2\hat{\imath}+\hat{\jmath})}{\sqrt{5}}$.

(b) Draw a contour diagram for the function with $z = 0, 1, 2$. Sketch the vector from the previous part on the contour diagram.

(c) Find the equation of the tangent plane at the point (1,2).

$-2(x - 1) + 1(y - 2) = z - 5$

(d) Find a vector that is tangent to the curve of the steepest ascent. (Note: This is a vector in $\mathbb{R}^3$, and it does NOT have to be a unit vector) $-2\hat{\imath} + \hat{\jmath} + 5\hat{k}$.

4. $f(x, y) = x^2 - 2xy + 3y^2 - 8y$

   (a) Find the gradient.

   $(2x - 2y)\hat{\imath} + (-2x + 6y - 8)\hat{\jmath}$

   (b) Find all critical points of f.

   $(2, 2)$

   (c) Find all second order partial derivatives of f.

   $f_{xx} = 2,\ f_{xy} = -2,\ f_{yy} = 6,\ f_{yx} = -2$

   (d) Identify each critical points as a local maximum, local minimum, saddle point where applicable.

   $(2, 2)$ is minimum.

5. $f(x, y) = z$ is a function whose contour is given on pg. 734, Fig 14.59. Suppose that $x(u, v) = v\sin(\pi u)$ and $y(v) = v^2$. We would like to compute the partial derivatives f_u and f_v when $u = -1$, and $v = 2$.

   (a) What is the value of x and y when $(u, v) = (-1, 2)$?

   $x = 0,\ y = 4$

   (b) Approximate the partial derivatives f_x and f_y at the point found in the previous problem.

   Answer may vary.

   (c) Approximate the partial derivatives f_u and f_v at $u = -1$ and $v = 2$.

   Answer may vary.