

AMS261 Practice Midterm 1

This is a practice exam for Midterm 1. The actual exam will be very similar to this exam.

You'll notice that there are many parts to each problems. These parts are intended as a guide to solving the actual problem at hand, and also to give you an idea as to how I'll be grading the midterm. A lot of the parts will not appear on the actual exam.

You may use a calculator for this exam. You must show work to receive full credit. An answer without any kind of work will not be credited, even if it is correct.

1. Let A and B be planes represented by equations $z = 2x - 2y + 5$ and $10 = -x + y - 4z$.
 - (a) Find a normal vector to plane A .
 - (b) Find a normal vector to plane B .
 - (c) Find the dot product between the normal vectors.
 - (d) Find the cross product between the normal vectors.
 - (e) What is the angle between plane A and plane B ?
 - (f) Find a vector that is parallel to the intersection of planes A and B .
 - (g) Find a plane perpendicular to plane A and plane B .
2. In class, we learned how to take the projection of one vector onto another vector. In this problem, you'll take the projection of one vector onto a plane. Let C be a plane given by the equation $50 = 2x - 4y + z$, and let $\vec{v} = 3\hat{i} + 2\hat{j} - 6\hat{k}$. Find the projection of \vec{v} onto plane A .
 - (a) Find a normal vector to the plane A
 - (b) Find the projection of vector v onto the normal vector found in the previous part.
 - (c) Find the projection of vector v onto the plane A . (Hint: Drawing a picture might be useful.)
3. There is a hiker climbing the surface $f(x, y) = 4 - x^2 + y$ at the point $(1, 2)$

- (a) The hiker wishes to climb the steepest ascent. What direction should the hiker move? (Note: Direction should be given as a unit vector.)
 - (b) Draw a contour diagram for the function with $z = 0, 1, 2$. Sketch the vector from the previous part on the contour diagram.
 - (c) Find the equation of the tangent plane at the point $(1,2)$.
 - (d) Find a vector that is tangent to the curve of the steepest ascent. (Note: This is a vector in \mathbb{R}^3 , and it does NOT have to be a unit vector)
4. $f(x, y) = x^2 - 2xy + 3y^2 - 8y$
- (a) Find the gradient.
 - (b) Find all critical points of f .
 - (c) Find all second order partial derivatives of f .
 - (d) Identify each critical points as a local maximum, local minimum, saddle point where applicable.
5. $f(x, y) = z$ is a function whose contour is given on pg. 734, Fig 14.59. Suppose that $x(u, v) = v \sin(\pi u)$ and $y(v) = v^2$. We would like to compute the partial derivatives f_u and f_v when $u = -1$, and $v = 2$.
- (a) What is the value of x and y when $(u, v) = (-1, 2)$?
 - (b) Approximate the partial derivatives f_x and f_y at the point found in the previous problem.
 - (c) Approximate the partial derivatives f_u and f_v at $u = -1$ and $v = 2$.