AMS261 Practice Final 2

Note: All oriented surfaces are given their "natural" orientation.

1. Let $f(x, y) = 9x^2 - 2x + 4y^2 - y$ and $R = \{\frac{x^2}{4} + \frac{y^2}{9} \leq 1\}$.
   (a) Does f necessarily have an absolute max or min on R? Explain.
   Yes.
   (b) Find all critical points of f in the interior of the region R.
   $(1/9, 1/8)$
   (c) Classify each of the critical points as relative maximum, minimum, or a saddle point where applicable.
   Relative Minimum
   (d) Find the absolute max/min value of f on region R.
   $-25/144, 41$
   (e) Let $R' = \{\frac{x^2}{4} + \frac{y^2}{9} \leq 1.0001\}$. Estimate the absolute max/min value of f on R'.
   $-25/144, 41.00385$

2. Let $\vec{F}(x, y, z) = yze^{xyz}\hat{\imath} + xze^{xyz}\hat{\jmath} + xye^{xyz}\hat{k}$ and let $C = \{x^2 + y^2 = a^2, z = 0\}$ oriented by the right hand rule with respect to $\hat{k}$.
   (a) Find the curl of $\vec{F}$.
   0
   (b) Is $\vec{F}$ conservative? Explain.
   Yes.
   (c) Find the work done by the $\vec{F}$ on a particle that starts from the point $(1, 0, 0)$ and goes around C exactly once along the orientation.
   0

3. Let $\vec{F}(x, y, z) = xz\hat{\imath} + yz\hat{\jmath} + \hat{k}$, let $S = \{x^2 + y^2 + z^2 = b^2, 0 \leq z\}$. Find the flux of $\vec{F}$ through S.
   $\pi b^2(b^2 + 2)/2$

4. Let $\vec{F}(x, y, z) = xyz\hat{k}$, let $S = \{z = 9 - y^2, 0 \leq y \leq 2, 0 \leq x \leq c\}$.
   (a) Find $\nabla \times \vec{F}$
   $xz\hat{\imath} - yz\hat{\jmath}$

(b) Find the flux of $\nabla \times \vec{F}$ through the surface S.
$-176c/5$

5. Let $\vec{F}(\vec{r}) = 2\vec{r}$

(a) Find the divergence of $\vec{F}$.
6

(b) Consider the following integral:

$$\int_0^b \int_0^{d/2} \int_0^{d-y} dx dy dz$$

where b and d are positive real numbers. Sketch the region of integration and evaluate the integral.
$3d^2b/8$

(c) Find the flux of $\vec{F}$ through the surface bounding the region in the previous part.
$9d^2b/4$