

AMS261 Practice Final 1

Note: All oriented surfaces are given their “natural” orientation.

1. Let $f(x, y) = 4x^3 - 6x^2 + 6y^2 + 5$ and $R = \{x^2 + y^2 \leq 9\}$.
 - (a) Does f necessarily have an absolute max or min on R ? Explain. Since f is continuous (because it is a polynomial), and R is closed and bounded, by the Extreme value theorem f has both absolute maximum and absolute minimum.
 - (b) Find all critical points of f in the interior of the region R . $(0, 0)$, $(1, 0)$
 - (c) Classify each of the critical points as relative maximum, minimum, or a saddle point where applicable. $(0, 0)$ is saddle. $(1, 0)$ is a rel min.
 - (d) Find the absolute max/min value of f on region R .
59
-157
 - (e) Let $R' = \{x^2 + y^2 \leq 8.9998\}$. Estimate the absolute max/min value of f on R' .
58.9988
-157.0048
2. Let $\vec{F}(x, y, z) = \frac{2z\hat{i} - 2x\hat{k}}{x^2 + z^2}$ and let $C = \{x^2 + z^2 = 1, y = 0\}$ oriented by the right hand rule with respect to \hat{j} .
 - (a) Find the curl of \vec{F} .
 $\vec{0}$
 - (b) Is \vec{F} conservative? Explain.
No.
 - (c) Find the work done by the \vec{F} on a particle that starts from the point $(0, 0, -1)$ and goes around C exactly once along the orientation.
 4π
3. Let $\vec{F}(\vec{r}) = \vec{r}$, let $S = \{x^2 + z^2 = b^2, 0 \leq y \leq c\}$. Find the flux of \vec{F} through S .
 $2b^2c\pi$

4. Let $\vec{F}(x, y, z) = e^{xyz}\hat{k}$, let S be the triangle formed by the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

(a) Find $\nabla \times \vec{F}$
 $(xz, -yz, 0)e^{xyz}$

(b) Find the flux of $\nabla \times \vec{F}$ through the surface S .
 0

5. Let $\vec{F}(x, y, z) = (z - x)\hat{i} + (z + y)\hat{j} - z^2\hat{k}$

(a) Find the divergence of \vec{F} .
 $-2z$

(b) Consider the following integral:

$$\int_{-5}^5 \int_{-4\sqrt{1-z^2/25}}^{4\sqrt{1-z^2/25}} \int_{-\sqrt{1-y^2/16-z^2/25}}^{\sqrt{1-y^2/16-z^2/25}} z dx dy dz$$

Sketch the region of integration and evaluate the integral.

Ellipsoid centered at the origin with axes of length 1, 4, and 5.

$$\int_{-5}^5 \int_{-4\sqrt{1-z^2/25}}^{4\sqrt{1-z^2/25}} \int_{-\sqrt{1-y^2/16-z^2/25}}^{\sqrt{1-y^2/16-z^2/25}} z dx dy dz = 0$$

(c) Find the flux of \vec{F} through the surface bounding the region in the previous part. 0