AMS261 Practice Final 1

Note: All oriented surfaces are given their "natural" orientation.

1. Let $f(x, y) = 4x^3 - 6x^2 + 6y^2 + 5$ and $R = \{x^2 + y^2 \leq 9\}$.

   (a) Does f necessarily have an absolute max or min on R? Explain.

   (b) Find all critical points of f in the interior of the region R.

   (c) Classify each of the critical points as relative maximum, minimum, or a saddle point where applicable.

   (d) Find the absolute max/min value of f on region R.

   (e) Let $R' = \{x^2 + y^2 \leq 8.9998\}$. Estimate the absolute max/min value of f on R'.

2. Let $\vec{F}(x, y, z) = \frac{2z\hat{i} - 2x\hat{k}}{x^2+z^2}$ and let $C = \{x^2 + z^2 = 1, y = 0\}$ oriented by the right hand rule with respect to $\hat{j}$.

   (a) Find the curl of $\vec{F}$.

   (b) Is $\vec{F}$ conservative? Explain.

   (c) Find the work done by the $\vec{F}$ on a particle that starts from the point $(0, 0, -1)$ and goes around C exactly once along the orientation.

3. Let $\vec{F}(\vec{r}) = \vec{r}$, let $S = \{x^2 + z^2 = b^2, 0 \leq y \leq c\}$. Find the flux of $\vec{F}$ through S.

4. Let $\vec{F}(x, y, z) = e^{xyz}\hat{k}$, let S be the triangle formed by the points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$.

   (a) Find $\nabla \times \vec{F}$

   (b) Find the flux of $\nabla \times \vec{F}$ through the surface S.

5. Let $\vec{F}(x, y, z) = (z - x)\hat{i} + (z + y)\hat{j} - z^2\hat{k}$

   (a) Find the divergence of $\vec{F}$.

   (b) Consider the following integral:

   $$\int_{-5}^{5} \int_{-4\sqrt{1-z^2/25}}^{4\sqrt{1-z^2/25}} \int_{-\sqrt{1-y^2/16-z^2/25}}^{\sqrt{1-y^2/16-z^2/25}} z\,dx\,dy\,dz$$

   Sketch the region of integration and evaluate the integral.

   (c) Find the flux of $\vec{F}$ through the surface bounding the region in the previous part.