Solution Guide to Practice Exams

1. The a two plane in $\mathbb{R}^3$ is completely determined by its normal vector and a point. (This is also true for a line in $\mathbb{R}^2$.

   Given two planes A and B the angle between them is equal to the angle between the normal vectors. If you aren't sure why that should be the case, try drawing a picture, with just lines instead of planes. (Drawing the two planes as lines is same as taking a perpendicular cross section of both of those planes.) You'll immediately see that the normal vectors form the same angle as the planes.

   There is a very simple way of measuring angle between two vectors. If you don't know how to do this, then I suggest going back to chapter 13.

   The second part of the problem is to find a plane that is perpendicular to planes A and B.

   Again, just as in the previous problem, you have to translate everything in terms of normal vectors. (Just replace the word "plane" with "vector")

2. We never talked about projecting a vector onto a plane in class. But so what? You should be able to apply what you learned to solve this problem

   Again, you should think about the normal vector to the plane. You should try drawing a picture of a vector, a plane and its normal vector, and try to figure out how projection of a vector onto a plane is related to... something we did talk about in class. It will help a lot if the vector you are drawing and the normal vector start from a common point that is on the plane.

3. Probably the hardest part of this problem is understanding the difference between the gradient vector of a function in two variables and the gradient vector of a function in three variables.

   You are given a function f that depends on x and y, just TWO variables. If you take the gradient of this function, you will get a vector that has TWO components, and this vector will live inside the xy-plane. This vector is NOT the normal vector to the SURFACE $f(x, y) = z$. It will be a normal vector to a CURVE on the xy-plane. (Can you guess what the curve is?)

If you want to get a normal vector to a SURFACE $f(x, y) = z$, you will need a function in THREE variables. THREE, NOT TWO!.

"But we only have a function in two variables, where do we get this function with three variables?" you might ask.

If you have no idea how to get a function in THREE variables out of a function in TWO variables, you should consult 12.5 of the textbook.

4. Come on folks. I went over this stuff on WEDNESDAY. That was TWO days ago.

   Just be careful when you take partial derivatives. And don't forget that a critical point is when either the gradient is UNDEFINED or ZERO. You MUST CHECK BOTH THINGS. UNDEFINED or ZERO. A LOT OF PEOPLE FORGET TO CHECK WHEN THE GRADIENT IS UNDEFINED. Know how to use the 2nd derivative test.

5. Remember how to estimate partial derivatives from a contour diagram? If you don't, go back to 14.1 and look at some examples or do some problems.

   This isn't JUST estimating partial derivatives though. You are shown a contour diagram of a function f, which depends on x and y. The x and y themselves are functions of some other variable(s). This sounds a lot like... the CHAIN RULE.

   The thing that most people get confused about the Chain rule is WHERE to evaluate those derivatives. Just remember that when you take the partial of f in the chain rule, you are treating it as a function of x and y, without thinking about the dependence of x and y on some other variable. (This is actually completely analogous to the 1-variable chain rule, but people get confused there too so... good luck?)