MAT 534: HOMEWORK 4

DUE FRI, SEP. 28, 2012

1. Classify all groups of order 75.

2. Classify all groups of order 20.

3. Let Q be the subgroup in $\mathbb{R}^n$ generated by elements of the form $e_i - e_j$, $i \neq j$, and let $P = \{\lambda \in \mathbb{R}^n \mid \sum \lambda_i = 0, \lambda \cdot \alpha \in \mathbb{Z}\ \forall \alpha \in Q\}$. (Here e_i are the standard generators of $\mathbb{Z}^n$: $e_i = (0, \dots, 1, \dots 0)$, with 1 in the i^{th} place, and $\lambda \cdot \alpha = \sum \alpha_i \lambda_i$ is the usual dot product in $\mathbb{R}^n$.)

Show that P, Q are free abelian groups of rank $n-1$. Show that $Q \subset P$ and describe the quotient P/Q.

4. Let G be the abelian group generated by three elements x, y, z with defining relations $x^3 = xy^2z^3 = 1$. Write this group in the canonical form (i.e., as a product of cyclic groups).

5. Same question for the group $\mathbb{Z}^3/L$, where L is the subgroup generated by elements

$$\begin{aligned} u_1 &= (3, 2, 5) \\ u_2 &= (0, 1, 3) \\ u_3 &= (0, 1, 5) \end{aligned}$$

6. Let $L \subset \mathbb{Z}^n$ be the subgroup generated by rows of an $n \times n$ matrix A with integer entries. Show that if $\det A = 0$, then $\mathbb{Z}^n/L$ is infinite, and if $\det A \neq 0$, then $|\mathbb{Z}^n/L| = |\det A|$.