MAT 141
ASSIGNMENT 4

DUE SEPT 27, 2005

Please write proofs (or at least some reasoning if you can't give a full proof), not just answers! So if a problem asks whether some set has a maximum, do not just write "No", write "No, it doesn't because..."

As usual, you are allowed to use all the theorems stated in the appropriate chapters of the book (whether or not we have discussed them in class).

Notation:

$\mathbb{Z}$ – set of integer numbers

$\mathbb{P}$ – set of positive integer numbers

$\mathbb{Q}$ – set of rational numbers

(1) Compute the following sum:

$$\sum_{i=1}^{n}(i+3)^2$$

(2) Let $F(n)$ be the n-th Fibonacci number, i.e. $F(1) = F(2) = 1$, and $F(n) = F(n-1) + F(n-2)$ for $n \geq 3$. Let $\phi = \frac{1+\sqrt{5}}{2}$ (this number is frequently called the *golden ratio*), and $\psi = \frac{1-\sqrt{5}}{2}$.

(a) Show that $\phi^2 = 1 + \phi$, $\psi^2 = 1 + \psi$

(b) Prove by strong induction that $F(n) = \frac{\phi^n - \psi^n}{\sqrt{5}}$.

(3) Section 1.7, problem 1

(4) Section 1.7, problem 2

(5) Section 1.11, problem 1

(6) Section 1.11, problem 2

(7) Section 1.15, problem 1